

# Hidden Testing and Selective Disclosure of Evidence

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## Abstract

A decision maker faces a choice to withdraw or retain a product but is uncertain about its safety. An agent can gather information through sequential testing. Players agree on the optimal choice under certainty, but the decision maker has a higher safety standard than the agent. We compare the case where testing is hidden and the agent can choose whether to disclose his findings to the case where testing is observable. The agent can exploit the additional discretion under hidden testing to his advantage if and only if the decision maker is sufficiently inclined to retain the product. Hidden testing then yields a Pareto improvement over observable testing if the conflict between players is larger than some threshold, but leaves the decision maker worse off and the agent better off if the conflict is smaller than this threshold.

Keywords: endogenous information acquisition, overt vs covert information acquisition, verifiable disclosure, mandatory vs voluntary disclosure, transparency, questionable research practices

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# 1 Introduction

Pharmaceutical companies have recently come under scrutiny for selective reporting of clinical trial outcomes.<sup>1</sup> As a response to demands for greater transparency, several companies have pledged to register trials and report their outcomes in open online databases (Goldacre et al. [2017]). At first sight, it seems that such transparency would improve regulation because pharmaceutical companies can no longer hide trials with unfavorable outcomes. However, companies may also strategically respond by running fewer trials and this could mean that the regulator has to base his decisions on weaker evidence.

This paper analyzes how transparent information acquisition affects the likelihood of correct regulatory decisions and welfare. A decision maker (DM, e.g. a regulator) has to choose whether to withdraw or retain a product based on evidence gathered by an agent (e.g. a company). However, there is a conflict of interest between the players: the DM is more averse to retaining an unsafe product than the agent, that is, the DM has a higher safety standard than the agent. The agent can sequentially run costless tests up to some deadline at which point the DM makes his choice. We first consider a setting of *observable testing*, in which the agent's findings are publicly observed. We contrast this with a setting of *hidden testing*, in which the DM neither observes for how long the agent has tested nor what the agent has found unless the agent chooses to disclose his findings.

One might expect that the DM is better off in equilibrium when testing is observable than when testing is hidden. After all, when testing is hidden, the agent can strategically hide evidence to influence the DM's choice. Provided the DM initially favors retaining the product, we show that the DM is indeed worse off under hidden testing if the conflict between players is small. However, if the conflict is large, then hidden testing yields a strict Pareto improvement over observable testing, that is, not only the agent but also the DM is strictly better off under hidden than observable testing.

This Pareto improvement arises for the following reasons. When testing is observable, the DM can react optimally to the evidence gathered, but the agent acquires evidence strategically. In particular, the agent can manipulate the DM's choice by stopping to test following certain findings while continuing to test following others. We characterize the unique equilibrium and show under which conditions the agent stops testing before the deadline even if further tests could be mutually beneficial. In particular, when the conflict is large and the DM initially favors retaining the product, the agent stops testing before the deadline if running further tests has the downside that the DM may come to favor withdrawal while the agent continues to favor retaining the product.

When testing is hidden, the agent can strategically withhold results from the DM. If the DM initially favors approval, it is indeed the case that the agent exploits this discretion to persuade the DM to retain the product more often than is optimal from the DM's perspective. However, since the agent has more influence over the DM's final choice than under observable testing, the agent

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<sup>1</sup>For example, GlaxoSmithKline was accused of having withheld data that suggested that its antidepressant Paxil was linked to suicidal behavior in teenagers (Rabin [2017]), and the Cochrane Review concluded that Roche withheld trial data on its influenza drug Tamiflu to make the drug look more effective (Jefferson et al. [2014]). See also Avorn [2006].

has more incentives to keep testing. Moreover, the agent reveals to the DM the findings of the additional tests run under hidden relative to observable testing if and only if these tests convince the agent that withdrawal is optimal. Therefore, the additional tests trigger the DM to withdraw instead of retaining the product if and only if withdrawal is in both players' interest, resulting in a mutual benefit.

By contrast, if the conflict of interest between players is small instead, the DM is worse off and the agent better off under hidden than observable testing. The reason is that the agent becomes equally informed under each regime, but under hidden testing, the agent's selective disclosure can still lead the DM to make suboptimal choices.

A Pareto improvement from hidden testing would also exist in a static setting in which the agent commits ex ante to how long he will test. We use the model with sequential testing to show that a Pareto improvement from hidden testing arises even if the agent has no commitment power.<sup>2</sup> Analyzing sequential testing also helps to illustrate the agent's trade-off more clearly as the agent is weighing up the advantages from testing versus stopping at each belief.

Our analysis can be applied to the context of postmarketing studies for medical drugs. These studies investigate a drug's safety or efficacy, or new indications of a drug after regulatory approval.<sup>3</sup> Frequently, these studies are initiated by companies themselves, but they can also be requested by the regulator as a condition of approval. However, companies often do not carry out the requested studies or do not release all their findings (Glasser et al. [2007]), and it has been argued that regulators lack the power to enforce compliance.<sup>4</sup> In light of this, demands for compulsory trial registration and reporting have been rising.

Our work predicts that registries help to improve regulatory decision making when the conflict of interest is small, but cause adverse welfare effects for all parties when the conflict of interest is large. In particular, without a trial registry, the company can run studies in private and withdraw the product if it finds that the product does not meet its own safety standards.<sup>5</sup> In case the company does not withdraw the product, the regulator infers that at least the company's own safety standards are being met and, hence, the regulator will not see a reason to withdraw the product. By contrast, with a trial registry, a company with a relatively low safety standard is discouraged from performing such tests. The reason is that these tests could yield weak evidence that the product is unsafe, sufficient for the regulator to take the product off the market but not sufficient to fail the company's safety standards. Therefore, the company does not run tests that could uncover serious safety concerns, harming both itself and the regulator.

Our results can also be applied in the context of academic publishing. As various replication

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<sup>2</sup>We assume that the agent's learning is limited by some exogenous resource constraints which take the form of a finite deadline in the dynamic setting. If the deadline were infinite, the agent would learn the state perfectly and the conflict of interest would not be of relevance. The finite deadline can also be interpreted as a limiting case of an infinite deadline with a convex cost of testing.

<sup>3</sup>The studies prior to approval usually only collect evidence on certain patient groups and outside routine clinical practice, e.g. see Smith et al. [2015].

<sup>4</sup>"The only thing the agency can do is take the drug off the market, which is a decision that often would not serve the public health very well.", Peter Lurie, deputy director of the Public Citizen's Health Research Group, available at: <https://www.citizen.org/news/fda-report-highlights-poor-enforcement-of-post-marketing-follow-up/>.

<sup>5</sup>The company may care about safety standards for liability reasons or to reputational concerns.

studies produced findings at odds with those in the initial studies, e.g. [Open Science Collaboration \[2015\]](#), calls for greater transparency in research increased, e.g. [Nosek et al. \[2018\]](#). One suggestion is to make registration of experiments and the use of result databases a condition for publication. However, as our paper points out, when all experiments need to be documented, a researcher with strong incentives to publish might be reluctant to further investigate a hypothesis that was supported by his existing experiments.<sup>6</sup> The reason is that these additional experiments could contradict his earlier findings and shed some doubt on whether the hypothesis is true. This may not be sufficient to deter the researcher from wanting to publish his findings, but the editor may no longer find the evidence convincing enough and reject the paper. If the researcher could experiment in private, he would discover whether or not additional experiments strongly reject the hypothesis. In this event, he will not seek publication, thereby reducing the likelihood that a false finding is published.

The paper is structured as follows. Section 2 outlines the model. Observable testing is analyzed in Section 3, hidden testing in Section 4. The welfare comparison can be found in Section 5. Extensions and robustness checks are in Section 6. A detailed comparison with the existing literature can be found in Section 7. Section 8 concludes. Proofs can be found in the appendix.

## 2 Model

Two players, a decision maker (she) and an agent (he), are uninformed about the state of the world  $\omega \in \{\omega_0, \omega_1\}$  and share a prior belief  $q_0 = \Pr(\omega_1) \in [0, 1]$ . Time is continuous and finite,  $t \in [0, T]$ . At time  $T$ , the decision maker (DM) chooses an action  $a \in \{a_0, a_1\}$  which affects the payoff of player  $i$  as follows,<sup>7</sup>

$u_i(a, \omega)$	$\omega_0$	$\omega_1$
$a_0$	$\frac{s_i}{1-s_i}$	0
$a_1$	0	1

where  $i \in \{DM, A\}$  and  $0 < s_A \leq s_{DM} < 1$ .<sup>8</sup> In particular, at any belief  $q = \Pr(\omega_1)$ , player  $i$  has a higher expected payoff from  $a_1$  than from  $a_0$  if and only if

$$\mathbb{E}_\omega u_i(a_0, \omega) = (1 - q) \frac{s_i}{1 - s_i} \leq q = \mathbb{E}_\omega u_i(a_1, \omega) \Leftrightarrow s_i \leq q. \quad (1)$$

Hence, if  $q < s_A$ , both prefer  $a_0$ , if  $s_A \leq q < s_{DM}$ , the DM prefers  $a_0$  while the agent prefers  $a_1$  and if  $s_{DM} \leq q$ , both prefer  $a_1$ . We refer to  $a_1$  as retaining and  $a_0$  as withdrawing the product and to  $s_i$  as player  $i$ 's safety standard.<sup>9</sup>

<sup>6</sup>The researcher may aim to advance science by supporting a hypothesis if and only if it is true, but be biased in favor of supporting a hypothesis if this increases his chances of publishing.

<sup>7</sup>This is without loss. Consider a general payoff function:  $u_i(a_0, \omega_0) = \alpha_i$ ,  $u_i(a_0, \omega_1) = \beta_i$ ,  $u_i(a_1, \omega_0) = \gamma_i$  and  $u_i(a_1, \omega_1) = \delta_i$ , where  $\alpha_i, \beta_i, \gamma_i, \delta_i \in \mathbb{R}$  and  $\alpha_i > \gamma_i$  and  $\delta_i > \beta_i$ . Then the results apply for  $s_i = \frac{\alpha_i - \gamma_i}{(\alpha_i - \gamma_i) + (\delta_i - \beta_i)}$ .

<sup>8</sup>For how the main result changes if  $s_A > s_{DM}$ , see Section 6.3.

<sup>9</sup>The model could also be applied to the approval process for new products in which case  $a_0$  would correspond to rejecting and  $a_1$  to approving the product.

When testing is observable, the agent runs publicly observable tests at no cost. When the agent tests, a signal arrives at random according to an exponential distribution with state-dependent arrival rate  $\lambda^\omega > 0$ . In particular, in state  $\omega$ , over an interval  $[t, t + dt)$ , a signal  $\theta = \omega$  is observed with probability  $\lambda^\omega dt$ , that is, the signal perfectly reveals the state. Let  $h^t$  denote the history up to time  $t$ . The (common) posterior belief  $q_t = Pr(\omega_1 | h^t)$  evolves as follows. If the agent tests and a signal arrives over the interval  $[t, t + dt)$ , then the belief jumps to 1 if  $\theta = \omega_1$  and jumps to 0 if  $\theta = \omega_0$ . If the agent tests and no signal arrives, the evolution of the posterior at time  $t$  is described by the following differential equation:<sup>10</sup>

$$\frac{dq_t}{dt} = -(\lambda^1 - \lambda^0) q_t (1 - q_t). \quad (2)$$

We assume  $\lambda^0 < \lambda^1$  which implies that the posterior belief drifts down when no signal arrives, that is, no news is bad news.<sup>11</sup> The agent's strategy selects to stop or to continue testing at time  $t$  conditional on  $h^t$  for each  $t \in [0, T]$ . The agent stops if he is indifferent between testing and stopping. In addition, once the agent stops, he cannot test again.<sup>12</sup> The DM's strategy selects an action choice  $a \in \{a_0, a_1\}$  at time  $T$  conditional on  $h^T$ . Since all payoff-relevant information contained in history  $h^t$  can be summarized by belief  $q_t$  for any  $t \in [0, T]$ , strategies can be thought of as mapping posterior beliefs into action choices.

When testing is hidden, the agent privately chooses when to stop and privately observes the arrival of signals.<sup>13</sup> Further, the agent decides whether to disclose all his findings to the DM or to remain silent. In particular, at time  $T$ , the agent's disclosure strategy selects message  $m$  conditional on  $h^T$ , where the set of feasible messages is given by  $M(h^T) = \{\emptyset, h^T\}$ . Then the DM's strategy selects an action choice  $a$  conditional on message  $m$ .

In either regime, the solution concept is a perfect Bayesian equilibrium. We impose the additional requirement that, for any off-path history, players update their beliefs about the state according to Bayes' rule using the testing technology.<sup>14</sup>

We restrict attention to situations in which the two players disagree on the optimal choice of  $a$  at some feasible posterior belief.<sup>15</sup> In particular, we assume the agent favors retention at the prior, i.e.  $s_A \leq q_0$ , and the DM favors withdrawal if testing until the deadline has not produced a signal,

<sup>10</sup>For details see Liptser and Shiryaev [2013].

<sup>11</sup>This news arrival process was chosen to illustrate the trade-offs clearly but the main insight does not hinge on this process as shown in Section 6.4. In the context of postmarketing studies, this process would represent that with some probability trials provide strong results about a drug's performance, while with some probability outcomes arise which give a weak indication that the drug is not performing well, e.g. the dropout rate may be higher among treated than untreated subjects and, as it is unclear if this occurred due to reasons unrelated to the drug's performance or if the treated subjects left because they experienced more severe side effects, such an outcome would cause the belief about the drug's performance to be revised down somewhat.

<sup>12</sup>These assumptions are chosen to avoid a multiplicity of equilibria that are payoff-equivalent except in a knife-edge case. More details are given in Sections 3 and 4.

<sup>13</sup>The key result remains unchanged if the DM could observe the time at which the agent stops. See also Footnote 29 after the results.

<sup>14</sup>This means off-path beliefs satisfy "no-signaling-what-you-don't-know" in the vein of Fudenberg and Tirole [1991].

<sup>15</sup>If the two players were to agree on the optimal action at all feasible posteriors, it is trivial that each player's payoff is the same under hidden and observable testing.

i.e.  $\underline{q} < s_{DM}$ , where

$$\underline{q} \equiv \frac{q_0 \exp(-\lambda^1 T)}{q_0 \exp(-\lambda^1 T) + (1 - q_0) \exp(-\lambda^0 T)}. \quad (3)$$

### 3 Observable Testing

When testing is observable, the players share a common posterior belief at any point in time. The agent can influence the DM's choice via controlling the flow of information: he can strategically choose to stop testing at certain posterior beliefs but continue testing at others. The agent may therefore stop testing if the DM is willing to retain the product. However, stopping leaves the agent worse off in the event that further evidence would have led both the DM and the agent to prefer withdrawal. In this section, we will characterize the unique equilibrium and show under which condition the agent stops testing for strategic reasons.

**Proposition 1.** *Under observable testing, an equilibrium exists and is unique. The equilibrium strategies are as follows. The DM chooses  $a_1$  if and only if  $s_{DM} \leq q_T$ . The agent stops at time  $t$  if a signal has arrived by time  $t$ . If no signal has arrived by time  $t$ , then*

1. for  $q_0 < s_{DM}$ , the agent keeps testing at time  $t$  for any  $t < T$ .
2. for  $q_0 \geq s_{DM}$ , there exists a critical level of the agent's standard  $\bar{s}_A(s_{DM}, q_0, T)$  such that
  - (a) if  $s_A > \bar{s}_A$ , the agent keeps testing at time  $t$  for any  $t < T$ , and
  - (b) if  $s_A \leq \bar{s}_A$ , the agent keeps testing at time  $t$  for any  $t < t^*$  and stops testing at  $t = t^*$ , where  $t^*$  satisfies

$$q_{t^*} = s_{DM} \Leftrightarrow t^*(s_{DM}, q_0) = \frac{\log\left(\frac{q_0}{1-q_0}\right) - \log\left(\frac{s_{DM}}{1-s_{DM}}\right)}{\lambda^1 - \lambda^0}. \quad (4)$$

The critical level  $\bar{s}_A \in (0, \underline{q})$  defined for  $q_0 \geq s_{DM}$  satisfies

$$\log\left(\frac{\bar{s}_A}{1-\bar{s}_A}\right) = \log\left(\frac{s_{DM}}{1-s_{DM}}\right) - \lambda^1 (T - t^*(s_{DM}, q_0)). \quad (5)$$

In equilibrium, given the DM's strategy, the agent anticipates that the product will be retained if and only if the belief at the deadline exceeds the DM's safety standard. If the DM is initially inclined to withdraw, i.e.  $q_0 < s_{DM}$ , the agent has an upside but no downside from testing. The upside is that a signal may arrive and, in this event, the agent has learned all there is to learn and the DM takes an action in line with the agent's interest. When no signal arrives, the belief drifts down. In this event, the DM continues to favor withdrawal and, consequently, the agent suffers no downside from testing.

By contrast, the agent has different concerns if the DM is initially inclined to retain the product, i.e.  $q_0 \geq s_{DM}$ . The agent has no downside from testing up until time  $t^*$ , at which point the belief has drifted down to the DM's safety standard if no signal has arrived. However, at time  $t^*$ , the

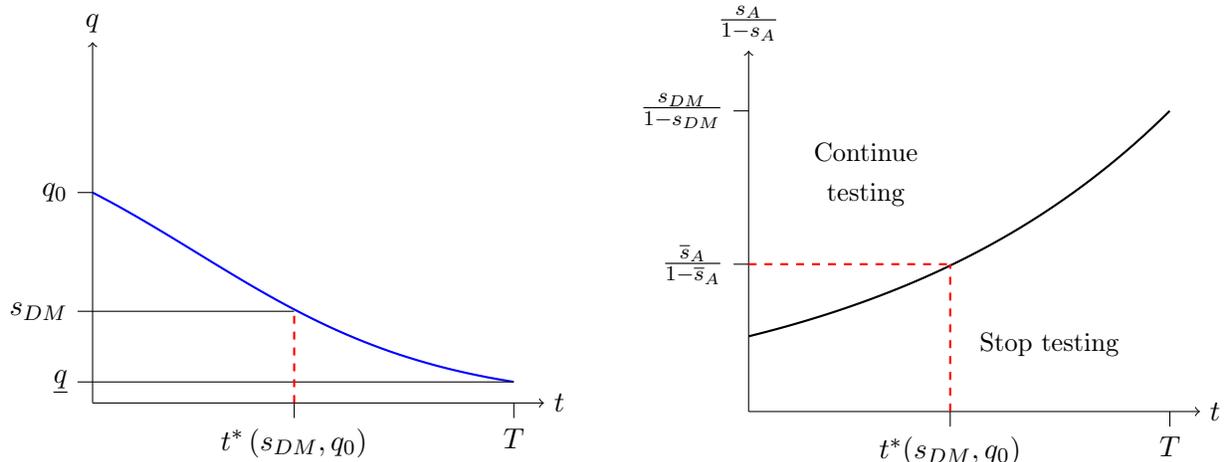


Figure 1: Left panel: The graph shows how the belief drifts down over time when no signal arrives. Time  $t^*(s_{DM}, q_0)$  is the time at which the belief reaches the DM's standard. Right panel: The graph divides levels of the agent's standard  $s_A$  into those for which the agent stops or continues testing at time  $t$  if  $q_t = s_{DM}$ . If the agent's standard is equal to the critical level  $\bar{s}_A$ , then the agent is indifferent between testing and stopping if  $q_t = s_{DM}$  at time  $t^*(s_{DM}, q_0)$ .

agent's decision to test is pivotal to the DM's choice even if no signal arrives. If the agent stops, the DM retains the product. If the agent continues and no signal arrives until the deadline, the DM withdraws the product instead.

Therefore, the agent faces a trade-off at time  $t^*$  provided the agent still favors retention when no signal has arrived by the deadline, i.e. provided  $s_A < \underline{q}$ .<sup>16</sup> By stopping at time  $t^*$ , the agent ensures that the DM retains the product, which is the agent's favored choice at this point. By testing further, the agent learns if a signal  $\theta = \omega_0$  arrives which causes the DM to withdraw the product. In this event, the agent benefits from having tested because, given the signal, the agent favors withdrawal. However, in the event that no signal arrives, the agent suffers from having tested because the DM withdraws although the agent favors retaining the product.<sup>17</sup> The downside from testing is stronger, the more the agent is inclined towards retention, i.e. the lower the agent's safety standard. The opportunity to wait for a signal  $\theta = \omega_0$  outweighs the downside from a suboptimal choice in case no signal arrives if and only if  $\bar{s}_A \leq s_A$ .<sup>18</sup> See also Figure 1.

The equilibrium is unique given the assumption that the agent stops if he is indifferent between stopping or testing. The agent is indifferent if a signal has arrived and further testing does not alter the DM's choice. Otherwise, the agent is only indifferent if  $s_A = \bar{s}_A$ ,  $t = t^*$  and no signal has arrived. In this knife-edge case, the equilibrium is not unique but we restrict attention to the equilibrium in which the agent stops in this situation.

<sup>16</sup>In the extreme case  $s_A = 0$ , the agent has a downside but no strict upside from testing at time  $t^*$ . This extreme case is closely related to Brocas and Carrillo [2007]'s framework when  $\pi = 0$  and  $\alpha < 1 - \beta$ .

<sup>17</sup>Note that for the agent to face a downside from testing, the posterior belief needs to drift down in the absence of a signal. If we instead assumed that the belief drifts up, i.e. no news is good news, the agent would always test and the DM could never do strictly better when testing is hidden. For an analysis of testing when the news arrival process features the belief drifting up or down over time see Section 6.4.

<sup>18</sup>The equilibrium payoff would be the same if the agent chose at time  $t = 0$  until which time  $t = k$  he will test for  $k \in [0, T]$ .

## 4 Hidden Testing

When testing is hidden, the agent can choose whether or not to disclose his findings. The agent's optimal disclosure depends on how the DM interprets no disclosure and, therefore, multiple equilibrium payoff profiles may exist. We aim to understand what happens when the agent exploits the additional discretion under hidden testing to his advantage, that is, we want to characterize agent-preferred equilibria. In the next section, we will then see under which conditions such strategic disclosure by the agent can benefit the DM.

**Proposition 2.** *Under hidden testing, an equilibrium always exists. There is a critical level of the DM's standard  $\bar{s}_{DM}(q_0, T)$  defined by*

$$\log\left(\frac{\bar{s}_{DM}}{1 - \bar{s}_{DM}}\right) = \log\left(\frac{q_0}{1 - q_0}\right) + \lambda^0 T, \quad (6)$$

such that the following are features of an agent-preferred equilibrium,

1. for  $s_A \geq \underline{q}$  or  $s_{DM} > \bar{s}_{DM}$ ,
  - the agent's testing strategy is as described in Proposition 1, the agent always discloses, i.e.  $m = h^T$  for every  $h^T$ ,
  - the DM chooses  $a_1$  if and only if the agent discloses  $h^T$  and the DM prefers  $a_1$  given  $h^T$ ,
2. for  $s_A < \underline{q}$  and  $s_{DM} \leq \bar{s}_{DM}$ ,
  - if no signal has arrived by time  $t$ , the agent keeps testing at time  $t$  for any  $t < T$ , the agent discloses  $h^T$  if and only if a signal  $\theta = \omega_0$  has arrived by time  $T$ ,
  - the DM chooses  $a_0$  if and only if the agent discloses  $h^T$  and the DM prefers  $a_0$  given  $h^T$ .

The proposition shows that the agent does not strictly benefit from suppressing findings in case he prefers withdrawing the product when no signal has arrived by the deadline, i.e.  $\underline{q} \leq s_A$ . Recall that we assume that the DM always prefers withdrawing the product in this event, i.e.  $\underline{q} < s_{DM}$ . Therefore, the agent can wait for a signal until the deadline, disclose his findings and the DM will act in the agent's interest whatever the agent found.

Consider the situation in which the agent favors retaining the product when no signal arrived by the deadline, i.e.  $\underline{q} > s_A$ . The agent would ideally keep testing for as long as no signal has arrived and then recommend retaining the product if and only if he himself favors retention given the evidence. To this end, suppose the agent were to suppress his findings if he prefers retention and to disclose his findings if he prefers withdrawal. If the agent favors withdrawal, then the players' interests are aligned and disclosing his findings will lead the DM to withdraw. If the agent suppresses his findings, the DM only infers that the agent favored retention and how the DM optimally reacts to this inference depends on the DM's safety standard  $s_{DM}$ .

If the DM is sufficiently inclined towards withdrawal, i.e. if  $s_{DM} > \bar{s}_{DM}$ , then the fact that the findings convinced the agent that retention is optimal is insufficient evidence for the DM to

retain the product. In particular, in the event that the agent does not disclose his findings, the DM's posterior rises to the critical level  $\bar{s}_{DM}$  but the DM optimally withdraws the product at this posterior. Therefore, the DM requires hard evidence to choose retention in equilibrium and the agent has no benefit from strategically suppressing information he has gathered. The agent effectively faces the same trade-offs as under observable testing.

By contrast, if the DM is sufficiently inclined towards retention, i.e.  $s_{DM} \leq \bar{s}_{DM}$ , then the DM is willing to retain the product based on the inference that the agent favors retention. In particular, although the DM does not know if the evidence led the agent to form a posterior that lies above or below the DM's safety standard, the DM finds it optimal to retain the product conditional on no disclosure. Hence, the agent's recommendation is indeed followed by the DM and the agent achieves his first-best payoff.<sup>19</sup>

The following proposition describes equilibrium payoffs across all equilibria under hidden testing.

**Proposition 3.** *Suppose testing is hidden.*

1. *If  $s_A \geq \underline{q}$  or  $s_{DM} > \bar{s}_{DM}$ , each player's payoff in the equilibrium in Part 1 of Proposition 2 is their unique equilibrium payoff.*
2. *If  $s_A < \underline{q}$  and  $s_{DM} \leq \bar{s}_{DM}$ , in any given equilibrium, each player's payoff is either equal to their payoff in the equilibrium in Part 1 or Part 2 of Proposition 2.*

The agent can always guarantee himself at least the same payoff as under observable testing by following the same testing strategy as under observable testing and then disclosing his findings. If the DM is sufficiently inclined to withdraw, i.e.  $s_{DM} > \bar{s}_{DM}$ , or if the players both favor withdrawal when no signal has arrived by the deadline, i.e.  $s_A \geq \underline{q}$ , then this is also the highest payoff the agent can achieve as shown by Proposition 2, implying that this is the unique equilibrium payoff.

Otherwise, i.e. if  $s_A < \underline{q}$  and  $s_{DM} \leq \bar{s}_{DM}$ , there are two possible profiles of equilibrium payoffs. If the DM withdraws the product following no disclosure in equilibrium, the agent achieves the same payoff as under observable testing. If the DM retains the product following no disclosure in equilibrium, the agent achieves his first-best payoff as in Part 2 of Proposition 2. These are the only two possible equilibrium payoffs because there cannot be an equilibrium in which the agent keeps testing when the belief has reached the DM's safety standard and the DM randomizes following no disclosure. This is because, if the DM were to retain the product with some probability strictly between 0 and 1 following no disclosure, the agent would keep testing until a signal arrives and suppress his findings if and only if no signal has arrived by the deadline since  $s_A < \underline{q}$ . But, in this event, the DM would strictly prefer withdrawing the product since  $\underline{q} < s_{DM}$ .<sup>20</sup>

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<sup>19</sup>If players' payoffs were discounted by the delay in decision making, the agent would send a message as soon as a signal arrives. This is because, following the arrival of a signal, the agent cannot learn more from running further tests and, by reporting sooner, guarantees that the DM chooses the mutually optimal action sooner which is strictly preferable for the agent. In equilibrium, following  $m = \emptyset$  at any time  $t > t^*$ , the DM would then infer that no signal arrived by time  $t$  and withdraw, leaving the agent to face the same trade-offs as under observable testing. However, an equilibrium without such unraveling would continue to exist when evidence accrues incrementally. See also Footnote 32.

<sup>20</sup>Proposition 3 would still apply if the agent chose at time  $t = 0$  until which time  $t = k$  he would test for  $k \in [0, T]$ .

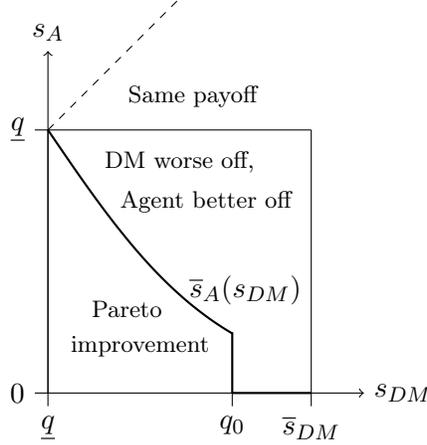


Figure 2: Payoff comparison between the agent-preferred equilibrium under hidden testing and the equilibrium under observable testing for combinations of  $s_{DM}$  and  $s_A$  with  $s_A \leq s_{DM}$  and  $\underline{q} < s_{DM}$ .

## 5 Welfare Comparison

This section will compare players' payoffs across hidden and observable testing and show under which conditions both players are strictly better off under hidden testing. As the previous section has shown, under hidden testing, the agent is never strictly worse off than under observable testing. However, it is unclear how the DM's payoff compares across regimes. Under observable testing, the DM can react optimally to the information generated by the agent's testing, but the agent may not exhaust all testing opportunities even if no signal has arrived. Under hidden testing, the agent exhausts all such testing opportunities but may strategically suppress findings which can hinder the DM from reacting optimally to the information gathered. Therefore, the DM faces a trade-off between hidden and observable testing.

**Proposition 4.** *In an agent-preferred equilibrium under hidden testing,*

1. *if  $s_A \geq \underline{q}$  or  $s_{DM} > \bar{s}_{DM}$ , each player has the same payoff as under observable testing,*
2. *if  $\bar{s}_A < s_A < \underline{q}$  or  $q_0 < s_{DM} \leq \bar{s}_{DM}$ , the agent has a strictly higher payoff and the DM a strictly lower payoff than under observable testing,*
3. *if  $s_A \leq \bar{s}_A$  and  $s_{DM} \leq q_0$ , each player has a strictly higher payoff than under observable testing.<sup>21</sup>*

The interesting payoff comparison occurs for parameter combinations for which the agent can strictly benefit from withholding information under hidden testing, that is, when the DM is sufficiently inclined to retain the product, i.e.  $s_{DM} \leq \bar{s}_{DM}$ , and players disagree on the optimal action when no signal has arrived by the deadline, i.e.  $s_A < \underline{q}$ .<sup>22</sup> See also Figure 2.

Then if the conflict between the two players is sufficiently small, i.e.  $\bar{s}_A < s_A$ , or the DM is initially inclined to withdraw, i.e.  $q_0 < s_{DM}$ , the agent keeps testing when no signal has arrived

<sup>21</sup>Note that  $\bar{s}_A < \underline{q}$  and  $q_0 < \bar{s}_{DM}$  so that three parts of Proposition 4 exhaust all possibilities.

<sup>22</sup>Recall that we assume throughout that  $\underline{q} < s_{DM}$ .

under either regime.<sup>23</sup> Therefore, under observable testing, the DM has access to all obtainable information and can react optimally. Hidden testing is then worse for the DM since the agent’s strategic suppression of findings leads her to retain the product in situations in which, knowing the full evidence, she would have chosen to withdraw the product.

**Corollary 1.** *The agent-preferred equilibrium under hidden testing yields a strict Pareto improvement over observable testing if and only if  $s_A \leq \bar{s}_A$  and  $s_{DM} \leq q_0$ .*

However, if the conflict between players is instead large, i.e.  $s_A \leq \bar{s}_A$ , and the DM initially favors retaining the product, i.e.  $s_{DM} \leq q_0$ , both can be strictly better off under hidden testing. Under observable testing, the agent stops testing when no signal has arrived by the time the belief has drifted down to the DM’s standard and the DM then retains the product. Indeed, in this situation, neither the DM nor the agent learns about whether a signal  $\theta = \omega_0$  would have arrived in the time remaining until the deadline. This is a mutual loss since both players would have preferred withdrawal in this event. By contrast, in an agent-preferred equilibrium under hidden testing, the agent will learn about whether a signal  $\theta = \omega_0$  arrives in the time remaining and he will disclose it truthfully. If no such signal arrives, the DM retains the product, just as she would have done under observable testing. Therefore, both players benefit from hidden testing.<sup>24</sup>

Proposition 4 has focused on the agent-preferred equilibrium under hidden testing. When considering the full range of equilibrium payoffs under hidden testing, the payoff comparisons of Proposition 4 hold in the weak sense.<sup>25</sup> This implies that hidden testing offers a Pareto improvement in a weak sense if and only if  $s_A \leq \bar{s}_A$  and  $s_{DM} \leq q_0$ , that is, no player is strictly worse off in any equilibrium under hidden testing than in the unique equilibrium under observable testing and there exists at least one equilibrium under hidden testing in which each player is strictly better off than under observable testing.<sup>26</sup>

## 6 Robustness and Extensions

The previous section has shown that both players can be better off under hidden than observable testing. This section will show what would happen if the DM could choose some intermediate regime in which the agent’s findings become public with some probability. Moreover, this section will show that a strict Pareto improvement also exists if the agent can only disclose signals under hidden testing (but not the absence of signals), if the conflict of interest is reversed and no news is good

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<sup>23</sup>Recall that, for the knife-edge case  $s_A = \bar{s}_A$ , we assume that the agent stops at time  $t^*$  under observable testing if he is indifferent between testing and stopping. If we instead assumed that the agent keeps testing at time  $t^*$  if indifferent, the case  $s_A = \bar{s}_A$  would be included in Part 2 instead of Part 3 of Proposition 4.

<sup>24</sup>If the agent had state-independent preferences, i.e.  $s_A \leq 0$ , then the equilibrium payoff profile under hidden testing would be unique and equal to the payoff profile under observable testing. The reason is that, if  $s_{DM} \leq q_0$ , the agent would not test at all under either regime as the DM chooses  $a_1$  without any tests. Moreover, if  $s_{DM} > q_0$ , there is no equilibrium under hidden testing in which the DM chooses  $a_1$  following  $m = \emptyset$  because, if the DM were to choose  $a_1$  following  $m = \emptyset$ , the agent would not test and send  $m = \emptyset$ . Hence, the agent faces the same trade-offs regarding testing under either regime.

<sup>25</sup>i.e. replace “strictly” with “weakly”.

<sup>26</sup>This follows from Part 2 of Proposition 3.

news or if the testing technology produces incremental evidence in favor of either state. Proofs for this section can be found in the supplementary appendix.

## 6.1 Intermediate regime

The DM never observes the agent's findings directly when testing is hidden, while the DM is guaranteed to observe the agent's findings when testing is observable. Could the DM benefit from a regime with an intermediate degree of transparency? Suppose the agent acquires evidence in private but, at time  $T$ , before the agent decides whether to disclose his findings, the DM observes the history  $h^T$  with probability  $p \in [0, 1]$ , independent of the agent's findings. If the DM does not observe the history, the agent chooses whether or not to disclose it.<sup>27</sup> For this exercise, we assume that the agent keeps testing if he is indifferent between testing and stopping.

**Proposition 5.** *If  $s_A < \bar{s}_A$  and  $s_{DM} \leq q_0$ , the probability  $p^*$  of observing  $h^T$  that maximizes the DM's payoff in the Pareto-best equilibrium is given by*

$$p^* = \frac{1 - \exp(-\lambda^0 (T - t^*(s_{DM}, q_0)))}{\exp(-\lambda^1 (T - t^*(s_{DM}, q_0))) \left(\frac{s_{DM}}{1-s_{DM}}\right) \left(\frac{1-s_A}{s_A}\right) - \exp(-\lambda^0 (T - t^*(s_{DM}, q_0)))} \quad (7)$$

where  $t^*(s_{DM}, q_0)$  is defined by (4) and  $p^* \in (0, 1)$ . Given  $p = p^*$ , the agent's payoff is equal to his payoff under observable testing.

When a Pareto improvement from hidden testing exists, the agent acquires more information under hidden testing because he can ensure that this additional information influences the DM's choice in line with his interests. However, to have an incentive to acquire more information, the agent only needs to have this advantage with a probability just high enough to make him indifferent between acquiring more information or not at time  $t^*$ . The larger is the conflict of interest, the lower must be the probability  $p^*$  with which the DM observes the evidence in order for the agent to keep testing at time  $t^*$  when no signal has arrived.<sup>28</sup>

## 6.2 Disclosure

The Pareto improvement from hidden testing continues to exist if we contrast observable testing with a game of hidden testing in which the agent can only disclose the arrival of a signal but not the absence of arrival.<sup>29</sup> To see this, note that, in the equilibrium described in Part 2 of Proposition 2, disclosure occurs if and only if a signal  $\theta = \omega_0$  arrives. The agent effectively recommends an action and the DM is willing to implement the recommended action. Therefore, the Pareto improvement

<sup>27</sup>Hence,  $p = 0$  corresponds to hidden testing and  $p = 1$  to observable testing.

<sup>28</sup>If  $s_A \geq \bar{s}_A$  or  $s_{DM} > q_0$ , the DM's payoff is maximized by  $p^* = 1$  because the agent never stops testing under observable testing when no signal has arrived and so the DM achieves her first-best payoff.

<sup>29</sup>Further, it is not crucial whether or not the time at which the agent stops testing is publicly observed. If it were publicly observed, the agent could always test until the deadline and then follow the same disclosure strategy as outlined in Part 2 of Proposition 2.

would also exist if the agent could merely send a cheap talk message or if the decision to retain or withdraw was delegated to the agent.<sup>30</sup>

### 6.3 Conflict of interest

If the conflict of interest were reversed, i.e.  $s_A > s_{DM}$ , no strict Pareto improvement from hidden testing would exist because the agent never has a reason to stop testing for strategic reasons when testing is observable. However, if  $s_A > s_{DM}$  and  $\lambda^1 < \lambda^0$ , that is, no news is good news, an analogous result to Corollary 1 applies: there exists some critical level of the agent's standard  $\tilde{s}_A$  such that the agent-preferred equilibrium under hidden testing yields a strict Pareto improvement over observable testing if and only if  $s_A \geq \tilde{s}_A$  and  $s_{DM} \geq q_0$ .

### 6.4 Incremental evidence

We have assumed that signals fully reveal the state. Does a Pareto improvement also exist if the testing technology generates only incremental evidence which can be in favor of either retention or withdrawal?<sup>31</sup> To show this, assume instead that testing generates a Brownian motion  $X$  with state-dependent drift, that is, we assume  $X$  starts at  $X_0 = 0$  and evolves according to

$$dX_t = \mu_\omega dt + dW_t, \quad (8)$$

where  $W$  is a standard Wiener process independent of  $\omega$  and  $\mu_0 = -1 < 0 < 1 = \mu_1$ .

Then, under observable testing, the (common) posterior belief at time  $t$ ,  $q_t = Pr(\omega_1|h^t)$ , depends only on  $X_t$ . It is useful to track posterior beliefs using the posterior log-likelihood ratio  $Q_t$ , where  $Q_t$  satisfies

$$Q_t = \log\left(\frac{q_t}{1-q_t}\right) = \log\left(\frac{q_0}{1-q_0}\right) + \log\left(\frac{\phi\left(\frac{X_t-t}{\sqrt{t}}\right)}{\phi\left(\frac{X_t+t}{\sqrt{t}}\right)}\right) = Q_0 + 2X_t, \quad (9)$$

and  $\phi$  is the density of a standard normal distribution. When testing is hidden, disclosing  $h^T$  is equivalent to disclosing the endpoint of the path  $X_T$  as the endpoint contains all payoff-relevant information the agent has. We impose no restrictions on  $s_A$ ,  $s_{DM}$ ,  $q_0$  and  $T$  other than  $s_A \leq s_{DM}$ . All remaining assumptions stay unchanged from Section 2.

**Proposition 6.** *Suppose testing generates a Brownian motion  $X$ . There exists a critical level of the agent's standard  $\hat{s}_A(s_{DM}, T)$  and a critical level of the DM's standard  $\hat{s}_{DM}(q_0, T, s_A)$  such that the agent-preferred equilibrium under hidden testing yields a strict Pareto improvement over observable*

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<sup>30</sup>If the agent were uncertain about  $s_{DM}$  and believed  $Pr(s_{DM} > \bar{s}_{DM}) > 0$ , then he would prefer to disclose a signal  $\theta = \omega_1$  and achieve retention for certain rather than not disclose and face a positive probability of withdrawal. The agent would then face the same trade-offs as under observable testing. However, if the DM could send a cheap talk message about whether or not  $s_A \leq \bar{s}_A$  and  $s_{DM} \leq q_0$ , Corollary 1 would still apply.

<sup>31</sup>In the original model, testing generates either conclusive evidence when a signal arrives or incremental evidence in favor of withdrawal when no signal arrives.

testing if  $s_A \leq \hat{s}_A$  and  $s_{DM} \leq \hat{s}_{DM}$ , where  $\hat{s}_A \in (0, s_{DM})$  is defined by

$$\log \left( \frac{\hat{s}_A}{1 - \hat{s}_A} \right) = \log \left( \frac{s_{DM}}{1 - s_{DM}} \right) + \log \left( 1 - \frac{\sqrt{T}}{\phi(\sqrt{T}) + \sqrt{T}\Phi(\sqrt{T})} \right), \quad (10)$$

and  $\hat{s}_{DM} \in (q_0, 1)$  is defined by

$$\log \left( \frac{\hat{s}_{DM}}{1 - \hat{s}_{DM}} \right) = Q_0 + \log \left( \frac{1 - \Phi \left( \frac{\frac{1}{2} \left( \log \left( \frac{s_A}{1 - s_A} \right) - Q_0 \right) - T}{\sqrt{T}} \right)}{1 - \Phi \left( \frac{\frac{1}{2} \left( \log \left( \frac{s_A}{1 - s_A} \right) - Q_0 \right) + T}{\sqrt{T}} \right)} \right), \quad (11)$$

where  $\phi$  is the density and  $\Phi$  the CDF of a standard normal distribution. Moreover, for any  $q_0$  and  $T$ , the region in which  $s_A \leq \hat{s}_A$  and  $s_{DM} \leq \hat{s}_{DM}$  is non-empty.

When testing is hidden, the agent-preferred equilibrium has a similar structure to the one in the original model: the agent discloses if he prefers withdrawal and does not disclose if he prefers retention. Conditional on the inference that the agent tested until the deadline and prefers retention, the DM is willing to retain the product.<sup>32</sup> Hence, the agent achieves his first-best payoff by strategically withholding information. Unlike in the original model, the critical level  $\hat{s}_{DM}$  of the DM's standard is a function of the agent's standard. This is because the agent's belief at the deadline can take any value in  $(0, 1)$  and, therefore, the fact that the agent prefers retention is weaker evidence in favor of state  $\omega_1$  if the agent is more inclined to retain the product, that is, if he has a lower safety standard.<sup>33</sup>

When testing is observable, the agent faces a trade-off between testing and stopping when the belief is equal to the DM's standard for the same reasons as in the original model. However, unlike in the original model, this event can arise with positive probability at any point in time. If the event  $q_t = s_{DM}$  arises shortly before the deadline, the agent's testing within the remaining time is relatively unlikely to move the belief below the agent's standard but relatively likely to move the belief above the agent's but below the DM's standard. Therefore, the downside of testing outweighs the upside. By contrast, earlier on in the game, when there is more time remaining until the deadline, the belief is more likely to drift towards the correct state and, therefore, it is less likely that testing leads players to disagree on the optimal choice. We show that there exists some critical time  $\hat{t}$  such that the agent prefers to keep testing in the event  $q_t \geq s_{DM}$  if and only if  $t < \hat{t}$ . This critical time  $\hat{t}$  decreases as the conflict between players increases, that is, as the agent's standard drops. See also Figure 3.

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<sup>32</sup>Note that, even if players' payoffs were subject to some discounting, there would exist an equilibrium under hidden testing in which  $m = \emptyset$  is followed by retention. At any time, the agent would face a trade-off between the action being taken sooner and being based on better information. Hence, if the agent continued until the deadline and reports  $m = \emptyset$ , the DM would infer that the agent did not find highly convincing evidence in favor of either retention or withdrawal earlier yet favors retention at the deadline. Based on this inference, the DM would retain the product if  $s_{DM}$  is sufficiently low.

<sup>33</sup>Since a history for which the players disagree always arises with positive probability, such an agent-preferred equilibrium exists for any  $s_A$ .

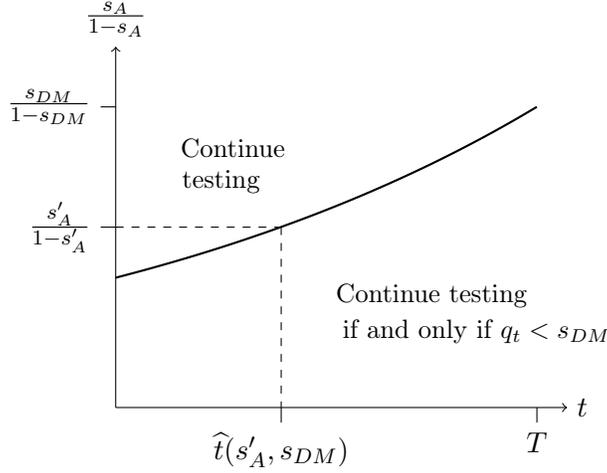


Figure 3: The graph shows the critical time  $\hat{t}$  for any given  $s_A$  holding  $s_{DM}$  fixed such that for all  $t < \hat{t}$ , the agent keeps testing at time  $t$  and, for all  $t \geq \hat{t}$ , the agent continues testing at time  $t$  if and only if  $q_t < s_{DM}$ .

Since the distribution over final beliefs is more complex than in the original model, it is harder fully characterize the equilibrium payoff comparison. However, to show that a Pareto improvement exists, it is sufficient to analyze a situation in which the agent's standard is low enough such that he would stop in the event  $q_t = s_{DM}$  under observable testing irrespective of when it occurs. If  $q_0 \geq s_{DM}$ , this implies that the belief never drops below the DM's standard and the DM retains the product with probability 1. Then hidden testing yields a Pareto improvement for the same reason as in the original model: both players benefit from hidden testing because the agent tests more under hidden testing and, if the additional tests lead to withdrawal being chosen, then both players prefer withdrawal.

Unlike in the original model, there are parameter combinations for which a strict Pareto improvement exists even if the DM is initially inclined to withdraw, i.e.  $q_0 < s_{DM}$ . The reason is that, when testing is observable, the agent stops testing if the belief rises to the DM's standard. Recall that this could never happen in the original model as the belief could jump up but not drift up. In the event that the agent stops at  $q_t = s_{DM}$  for  $t < T$ , hidden testing yields a mutual advantage as described above. However, with incremental evidence, hidden testing now also yields a disadvantage for the DM in the event that the belief never reaches the DM's standard over the time horizon  $[0, T]$  and lies above the agent's standard at time  $T$ . In this event, the DM prefers withdrawal but only chooses withdrawal if testing is observable. As the proof shows, if the DM is willing to implement the agent's recommended choice under hidden testing, i.e. if  $s_{DM} \leq \hat{s}_{DM}$ , then the benefit of hidden testing must outweigh the cost for the DM.

## 7 Related literature

Our work builds on the extensive literature on strategic information acquisition in sender-receiver games. Existing work has shown that, when incentives are misaligned, the DM values commitment

to take ex-post suboptimal actions to improve information acquisition e.g. Szalay [2005], and, for this reason, limiting transparency can be beneficial, e.g. Prat [2005]. In our paper, hidden testing has features of a limited liability insurance issued by the DM to the agent, but the fact that the DM benefits from hidden testing is in contrast with findings on limited liability insurance by Mackowiak and Wiederholt [2012].

In a cheap talk context, Argenziano et al. [2016] study how different transparency regimes and allocations of decision rights affect costly information acquisition. Deimen and Szalay [2019] compare delegation and communication when the agent can acquire different types of information and the degree of conflict arises endogenously.

Our focus is the verifiable disclosure of information. Unlike in optimal persuasion, e.g. Kamenica and Gentzkow [2011], the agent in our setting faces a given testing technology and only has the flexibility to continue or to stop acquiring information. Brocas and Carrillo [2007] also study how an agent exerts influence via controlling the flow of information, but they assume the agent has state-independent preferences and find that payoffs are equal across hidden and observable testing. Other related work also assumes the agent's preferences are state-independent. Henry and Ottaviani [2019] and McClellan [2020] compare payoffs under different forms of commitment when an agent controls a flow of public information, testing is costly and players' payoffs are discounted. Che et al. [2021] study persuasion when an agent chooses the directionality of public Poisson signals to influence a DM over time.<sup>34</sup> Assuming testing is hidden and costly, Felgenhauer and Schulte [2014] study persuasion when the agent can selectively disclose individual signals.

That incentives to acquire costly information can be higher under voluntary rather than mandatory disclosure is shown by e.g. Matthews and Postlewaite [1985], Farrell [1985] and Shavell [1994]. However, in our paper, the reason the agent acquires more information is not to avoid the skeptical inference that he is hiding unfavorable outcomes.

Polinsky and Shavell [2012] also show that social welfare can be higher under voluntary rather than mandatory disclosure in a model with transfers. They study a firm that can acquire costly information about product harm to increase the gains from trade with consumers. The firm benefits from additional information under both mandatory and voluntary disclosure but is willing to pay a higher cost for information acquisition if disclosure is voluntary. By contrast, in our paper, the agent acquires less information under observable than under hidden testing even though information is costless. In addition, our paper shows that that social welfare can increase in the sense of a strict Pareto improvement in a sender-receiver game with no transfers.

Previous work on sender-receiver games with no transfers produced different welfare results, in particular, that hidden testing always benefits the DM but harms the agent. In particular, Henry [2009] studies an agent whose ideal action choice differs from the DM's ideal action by a constant amount independent of the state. The agent commits to a quantity of costly research ex ante. If his choice is hidden, the agent conducts more research to have a larger pool of signals to select from but interestingly, this leaves the DM better informed since he can infer all signals due to unraveling

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<sup>34</sup>Nikandrova and Panes [2018], Che and Mierendorff [2019] and Mayskaya [2019] also use a Poisson testing technology but focus on optimal stopping by a single player.

in the vein of Milgrom [1981] and Grossman [1981].

Moreover, Felgenhauer and Loerke [2017] study an agent who desires approval independent of the state. The agent tests sequentially and can decide how informative each test will be. Surprisingly, they find that the agent runs only a single test in any Pareto-undominated equilibrium, whether testing is observable or hidden. However, if testing is hidden the agent runs a more informative test, because this makes it credible that he will not run further tests even if the outcome is unfavorable.<sup>35</sup>

In these papers, when disclosure is voluntary, the agent would benefit from the power to commit to disclosing all his findings as this would allow him to restore the outcome under observable testing. In our paper, the agent has the option to credibly disclose all his findings, yet a situation can arise in which the DM strictly benefits from hidden testing due to players having partially aligned interests.

In the context of medical trials, Dahm et al. [2009] find that a compulsory trial registry combined with a voluntary results database implements full transparency, but deters investment in research relative to a voluntary results database alone.<sup>36</sup> Further, Dahm et al. [2018] show that stricter monitoring of firms' reporting discourages information acquisition. They find that imperfect enforcement of mandatory disclosure may be optimal, e.g. if, at some intermediate level of monitoring, firms run trials but only some disclose negative findings, then, with stricter monitoring, some of these firms may stop running trials at all. We find that intermediate transparency is optimal for the DM even if tailored to a single agent. If the agent does not become sufficiently informed when transparency is high, the DM should grant the agent just enough discretion to make the agent indifferent between supplying more information or not.

Despite the agent facing different incentives for information acquisition and disclosure, our results share with Che and Kartik [2009] that the DM may be better informed when the agent is more biased.<sup>37</sup> Studying delegation to exogenously informed agents, Li and Suen [2004] find that more biased agents are delegated to more often. Moreover, Di Tillio et al. [2017] show that various welfare effects including a Pareto improvement can arise from manipulation of an RCT when the agent is privately informed ex ante.<sup>38</sup> Di Tillio et al. [2021] study more generally for which evidence distributions a selected sample is more informative than a random sample. Their work shares with this paper the feature that the agent's selective disclosure is informative for the DM but, in our paper, this is due to partially aligned interests rather than distributional properties.

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<sup>35</sup>Libgober [2020] shows that a receiver may be better off if the sender cannot verifiably disclose certain dimensions of an experiment because it may lead the sender to choose a different experiment that is more desirable along observed dimensions.

<sup>36</sup>Yoder [2021] studies incentives for research when tests are pre-registered and rewards can be conditioned on either the distribution of test outcomes or the outcome realization. Gall and Maniadis [2019] analyze a tournament between researchers with and without certain possibilities of inflating outcomes at a cost and find that preventing researchers from selective reporting also discourages more severe questionable research practices such as fabrication.

<sup>37</sup>See also Gerardi and Yariv [2008] and Dur and Swank [2005].

<sup>38</sup>Janssen [2018] studies a perfectly informed agent who tests sequentially to seek approval from the DM and finds that the DM can never strictly benefit from hidden testing.

## 8 Conclusion

This paper studies a DM facing a choice to retain or withdraw a product when uncertain about its safety. An agent can acquire information through sequential testing, but the agent's safety standard lies below the DM's. How is welfare affected when testing is hidden and the agent chooses whether or not to disclose his findings compared to when testing is observable? Our key contribution is to identify conditions under which hidden testing yields a strict Pareto improvement over observable testing. Under these conditions, when testing is observable, the agent does not exhaust all tests to become informed even though he bears no direct costs for conducting these tests. The reason for stopping is that additional information could lead the DM to change her action choice against the agent's interest. When testing is hidden, the agent can exploit the fact that their interests are partially aligned and strategically withhold evidence such that the DM acts in the agent's interest. However, due to this additional discretion under hidden testing, the agent has a reason to become better informed and, therefore, not only the DM but also the agent benefit from hidden testing.

While the current paper analyzes the interaction between a DM and an agent, there are other interesting consequences of transparency when several decision makers use the evidence as a basis for their choices or when several agents supply evidence. In particular, interesting externalities between agents arise, e.g. if an agent with a higher standard than the DM is added to the game. Then the agent with the higher standard would continue to supply evidence if the belief has dropped to the DM's standard, encouraging the agent with the lower standard to keep testing as well. These and related questions can be explored in future research.

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## A Appendix

### A.1 Proof of Proposition 1

We start by showing that the agent’s strategy is the unique best reply.<sup>39</sup> The agent always stops testing the first time a signal arrives because any further testing does not influence the DM’s belief and final action choice  $a$ . Suppose no signal arrived by time  $t$ .

Part 1.) We first show that the agent keeps testing at time  $t$  if  $0 < q_t < s_{DM}$  for any  $t < T$ . Let  $V_t(q, \tau)$  denote the agent’s expected continuation value at time  $t$  and posterior  $q$  when he stops the first time a signal arrives or at time  $\tau$ . If  $\tau = t$ , then given the DM’s strategy,

$$V_t(q_t, t) = (1 - q_t) \frac{s_A}{1 - s_A}. \quad (\text{A.1})$$

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<sup>39</sup>Recall that we assume that the agent stops if he is indifferent between testing and stopping and that once the agent stops he cannot test again.

If instead  $\tau = t' > t$ , then given the DM's strategy,

$$\begin{aligned} V_t(q_t, t') &= (1 - q_t) \frac{s_A}{1 - s_A} \Pr(q_{t'} < s_{DM} | q_t, \tau = t', \omega_0) + q_t \Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_1) \\ &= (1 - q_t) \frac{s_A}{1 - s_A} + q_t \Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_1), \end{aligned} \quad (\text{A.2})$$

where  $\Pr(q_{t'} < s_{DM} | q_t, \tau = t', \omega)$  is the probability that  $q_{t'} < s_{DM}$  at time  $t'$  conditional on the belief being  $q_t$  at time  $t$ , the agent stopping the first time a signal arrives or at time  $t'$  and the state being  $\omega$ . The agent keeps testing at any time  $t$  if  $0 < q_t < s_{DM}$  since

$$V_t(q_t, t') - V_t(q_t, t) > 0 \Leftrightarrow q_t \Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_1) > 0.$$

Then, given  $q_0 < s_{DM}$ , if no signal has arrived by time  $t$ , then  $0 < q_t < s_{DM}$  for any  $t$  and, hence, the agent keeps testing at any  $t < T$ .

Part 2.) Time  $t^*$  is the time at which  $q_t = s_{DM}$  if no signal has arrived by time  $t$ . By Bayes' rule:

$$s_{DM} = \frac{q_0 \exp(-\lambda^1 t^*)}{q_0 \exp(-\lambda^1 t^*) + (1 - q_0) \exp(-\lambda^0 t^*)} \Leftrightarrow \exp((\lambda^1 - \lambda^0) t^*) = \frac{q_0}{1 - q_0} \frac{1 - s_{DM}}{s_{DM}} \Leftrightarrow (4)$$

Given  $q_t > s_{DM}$ , we first show that the agent keeps testing at any time  $t < t^*$ . Suppose the agent stops testing the first time a signal arrives or at time  $\tau = t'$  where  $t' \leq t^*$ . If no signal has arrived by time  $t'$ , then  $s_{DM} \leq q_{t'} < 1$ . Clearly,

$$V_t(q_t, t') = (1 - q_t) \frac{s_A}{1 - s_A} (1 - \exp(-\lambda^0 (t' - t))) + q_t \quad (\text{A.3})$$

increases in  $t'$ . Hence, for  $q_0 > s_{DM}$ , when no signal has arrived, the agent keeps testing at any  $t < t^*$ . Next, consider  $q_{t^*} = s_{DM}$  at time  $t^*$ . Suppose the agent stops the first time a signal arrives or at time  $\tau = t'$  where  $t' > t^*$ . Clearly,

$$V_{t^*}(s_{DM}, t') = (1 - s_{DM}) \frac{s_A}{1 - s_A} + s_{DM} (1 - \exp(-\lambda^1 (t' - t^*))), \quad (\text{A.4})$$

increases in  $t'$ . Hence, if  $\tau > t^*$ , then  $\tau = T$  is optimal. Therefore, at time  $t^*$ , the agent chooses  $\tau = t^*$  and receives  $V_{t^*}(s_{DM}, t^*) = s_{DM}$  rather than choosing  $\tau = T$  and receiving (A.4) evaluated at  $t' = T$  if and only if

$$V_{t^*}(s_{DM}, t^*) - V_{t^*}(s_{DM}, T) \geq 0 \Leftrightarrow \log\left(\frac{s_A}{1 - s_A}\right) \leq \log\left(\frac{s_{DM}}{1 - s_{DM}}\right) - \lambda^1 (T - t^*), \quad (\text{A.5})$$

i.e. the agent stops at  $t^*$  if and only if  $s_A \leq \bar{s}_A$  using the definition of  $\bar{s}_A$  in (5). Clearly,  $\bar{s}_A > 0$  for finite  $T$  given (5). Further,  $\bar{s}_A < \underline{q}$  since  $\bar{s}_A$  decreases in  $s_{DM}$  given (4) and (5) and, if  $s_{DM}$  is at its lower bound, i.e.  $s_{DM} = \underline{q}$ , then  $t^* = T$  and  $\bar{s}_A = \underline{q}$ .

The DM's strategy is uniquely optimal given (1) independent of the agent's strategy for  $q_T \neq s_{DM}$ . Although the DM is indifferent between  $a_1$  and  $a_0$  at  $q_T = s_{DM}$ , there can be no equilibrium

in which the DM randomizes at  $q_T = s_{DM}$ . If the DM were to do so, the agent would strictly prefer to stop the first time that the belief is equal to the lowest belief at which the DM chooses  $a_1$  with probability 1 which is not well-defined.

## A.2 Proof of Proposition 2

Part 1) We first show that an equilibrium with these features exists for all parameter configurations. The DM's strategy is optimal since  $m = \emptyset$  does not arise on the equilibrium path and we can assign a belief such that  $\Pr(\omega_1|m = \emptyset) < s_{DM}$ . Further, the agent's disclosure strategy is optimal for any  $h^T$  for the following reason. If  $\Pr(\omega_1|h^T) \geq s_{DM}$ , then the DM chooses  $a_1$  given  $m = h^T$  and the agent prefers  $a_1$  since  $s_{DM} \geq s_A$ . If  $\Pr(\omega_1|h^T) < s_{DM}$ , both  $m = h^T$  and  $m = \emptyset$  lead to  $a_0$ . Given  $m = h^T$  for all  $h^T$ , the agent's optimal testing strategy is given by Proposition 1.

Next, we show that this is an agent-preferred equilibrium if  $s_A \geq \underline{q}$ . Since  $\underline{q} > \bar{s}_A$ , the agent keeps testing at any  $t$  if and only if no signal has arrived by time  $t$  for  $t < T$  by Proposition 1. Then  $m = h^T$  leads to the agent's favored action  $a$  for any feasible  $h^T$  since  $\Pr(\omega_1|h^T) \in \{0, \underline{q}, 1\}$  and  $1 > s_{DM} \geq s_A \geq \underline{q} > 0$ . Therefore, the agent achieves his first-best payoff and the equilibrium must be agent-preferred.<sup>40</sup>

Finally, we show that this is an agent-preferred equilibrium if  $s_A < \underline{q}$  and  $s_{DM} > \bar{s}_{DM}$ . We start by showing that, in any equilibrium, the DM must choose  $a_0$  following  $m = \emptyset$ . Suppose the DM chose  $a_1$  following  $m = \emptyset$  with probability  $p \in (0, 1)$ . Then, the agent's optimal disclosure would be to choose  $m = \emptyset$  if and only if  $s_A \leq \Pr(\omega_1|h^T) < s_{DM}$ . Given  $s_{DM} > \bar{s}_{DM}$  and  $\bar{s}_{DM} > q_0$  by (6), the agent would keep testing if no signal has arrived by time  $t$  for any  $t < T$  since stopping at time  $t$  would yield

$$V_t(q_t, t) = (1 - q_t) \frac{s_A}{1 - s_A} (1 - p) + q_t p. \quad (\text{A.6})$$

while testing until a signal arrives or  $\tau = t' > t$  would yield

$$\begin{aligned} V_t(q_t, t') &= (1 - q_t) \frac{s_A}{1 - s_A} (1 - \exp(-\lambda^0(t' - t)) + \exp(-\lambda^0(t' - t))(1 - p)) \\ &\quad + q_t (1 - \exp(-\lambda^1(t' - t)) + \exp(-\lambda^1(t' - t))p), \end{aligned} \quad (\text{A.7})$$

and for any  $t' > t$ ,

$$\begin{aligned} V_t(q_t, t') &> V_t(q_t, t) \\ \Leftrightarrow (1 - q_t) \frac{s_A}{1 - s_A} p (1 - \exp(-\lambda^0(t' - t))) + q_t (1 - p) (1 - \exp(-\lambda^1(t' - t))) &> 0. \end{aligned} \quad (\text{A.8})$$

But given the agent's strategy, the DM's consistent belief given  $m = \emptyset$  would satisfy  $\Pr(\omega_1|m = \emptyset) = \underline{q}$  and the DM should choose  $a_0$  with probability 1 since  $\underline{q} < s_{DM}$ , leading to a contradiction.

Next, suppose the DM chose  $a_1$  following  $m = \emptyset$  with probability 1. Then, the agent would choose  $m = h^T$  if signal  $\theta = \omega_0$  arrived and  $m = \emptyset$  otherwise.<sup>41</sup> The agent would keep testing at  $t$  if

<sup>40</sup>The agent's first-best payoff is the payoff the agent would achieve if he was also in control of taking action  $a$ .

<sup>41</sup>If the agent chose  $m = \emptyset$  if and only if no signal arrived, the DM's consistent belief given  $m = \emptyset$  would satisfy

and only if no signal has arrived by time  $t$  for  $t < T$  since (A.8) holds for  $p = 1$ . Given the agent's strategy, the DM's consistent beliefs given  $m = \emptyset$  would satisfy

$$\frac{\Pr(\omega_1|m = \emptyset)}{\Pr(\omega_0|m = \emptyset)} = \frac{q_0}{1 - q_0} \frac{1}{\exp(-\lambda^0 T)} \Leftrightarrow \log\left(\frac{\Pr(\omega_1|m = \emptyset)}{\Pr(\omega_0|m = \emptyset)}\right) = \log\left(\frac{q_0}{1 - q_0}\right) + \lambda^0 T, \quad (\text{A.9})$$

and, hence,  $\Pr(\omega_1|m = \emptyset) = \bar{s}_{DM}$  by (6). But then the DM should choose  $a_0$  given these beliefs since  $\bar{s}_{DM} < s_{DM}$ , leading to a contradiction. Given that the DM's unique equilibrium strategy involves choosing  $a_0$  following  $m = \emptyset$ , the agent's strategy in the equilibrium of Part 1 is his best reply and the equilibrium is agent-preferred.

Part 2) The DM's consistent beliefs given  $m = \emptyset$  satisfy (A.9) and, hence,  $\Pr(\omega_1|m = \emptyset) = \bar{s}_{DM}$  by (6). Since  $\bar{s}_{DM} \geq s_{DM}$ , the DM optimally chooses  $a_1$  following  $m = \emptyset$ . In addition, the agent has no reason to deviate because he achieves his first-best payoff. This implies that the equilibrium is agent-preferred.

### A.3 Proof of Proposition 3

Part 1) The agent can always achieve at least the same payoff as in the equilibrium of Part 1 of Proposition 2 by following the testing strategy in Proposition 1 and reporting  $m = h^T$  for any  $h^T$ .<sup>42</sup> Since this equilibrium is agent-preferred by Part 1 of Proposition 2, this must be the agent's unique equilibrium payoff. If the agent's equilibrium payoff is unique, then the DM's equilibrium payoff must also be unique.

Part 2) First, if the DM chooses  $a_0$  following  $m = \emptyset$  in equilibrium, the payoff profile must be equal to the one in the equilibrium in Part 1 of Proposition 2. This is because the agent's optimal disclosure must result in the DM choosing  $a_1$  if  $s_{DM} \leq \Pr(\omega_1|h^T)$ , because then also  $s_A \leq \Pr(\omega_1|h^T)$  and, otherwise, any message results in  $a_0$ . Therefore, by Proposition 1, the agent's optimal testing strategy is unique and as described in Proposition 1.

Second, if the DM chooses  $a_1$  following  $m = \emptyset$  in equilibrium, the payoff profile must be equal to the one in the equilibrium in Part 2 of Proposition 2. This is because the agent's optimal disclosure must result in the DM choosing the agent's preferred action for any  $h^T$  because, if  $\Pr(\omega_1|h^T) < s_A$ , the agent reports  $m = h^T$  and the DM chooses  $a_0$  and, otherwise, the agent reports  $m = \emptyset$  and the DM chooses  $a_1$ . Therefore, by Proposition 2, the agent's testing strategy is unique and as described in Part 2 of Proposition 2.

Finally, no other equilibrium payoff would arise if the DM chose  $a_1$  following  $m = \emptyset$  with some probability  $p \in (0, 1)$ . Suppose the DM chose  $a_1$  following  $m = \emptyset$  with some probability  $p \in (0, 1)$ . Then the agent chooses  $m = \emptyset$  if and only if no signal arrived by time  $T$  and  $s_A \leq \Pr(\omega_1|h^T) < s_{DM}$ . If  $q_0 \geq s_{DM}$ , the agent keeps testing at time  $t$  if and only if no signal has arrived by time  $t$  for  $t < t^*$  by Proposition 1. If  $q_0 \geq s_{DM}$  and the agent optimally stops at time  $t^*$ , the equilibrium payoff is as in Part 1 of Proposition 2. If  $q_0 \geq s_{DM}$  and the agent does not stop at time  $t^*$  or if  $q_0 < s_{DM}$ , the DM's consistent beliefs given  $m = \emptyset$  would lead her to choose  $a_0$  with probability 1 by the same

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$\Pr(\omega_1|m = \emptyset) = q$ , leading to a contradiction by the same reasoning as above.

<sup>42</sup>Recall that we assume that the DM's beliefs following  $m = h^T$  must be consistent with the testing technology.

reasoning as in the proof of Part 1 of Proposition 2. With this we have exhausted all possibilities for the DM's action following  $m = \emptyset$ .

#### A.4 Proof of Proposition 4

Part 1) This is implied by Part 1 of Proposition 2.

Part 2 and 3), i.e.  $s_A < \underline{q}$  and  $s_{DM} \leq \bar{s}_{DM}$ . The agent must be strictly better off in the agent-preferred equilibrium under hidden testing than under observable testing. This is because, by Proposition 3, the agent's payoff under hidden testing is either equal to the payoff under observable testing or equal to the payoff in the equilibrium in Part 2 of Proposition 2. The equilibrium in Part 2 of Proposition 2 is agent-preferred by Proposition 2. Hence, the agent is strictly better off in the agent-preferred equilibrium under hidden testing than under observable testing.

By Propositions 1 and 2, the DM's payoff under hidden testing is the same as under observable testing if a signal arrives in the interval  $[0, t^*]$ , where  $t^*$  is the time at which  $q_t = s_{DM}$  when no signal has arrived by time  $t$ , as defined in (4). Suppose no signal has arrived by  $t^*$ . Since  $s_{DM} \leq \bar{s}_{DM}$ , under hidden testing, the DM chooses  $a_0$  if and only if  $\theta = \omega_0$  arrives over time horizon  $(t^*, T]$ . Therefore, the DM's expected payoff under hidden testing at time  $t^*$  when no signal has arrived is

$$s_{DM} + (1 - s_{DM}) (1 - \exp(-\lambda^0 (T - t^*))) \frac{s_{DM}}{1 - s_{DM}}. \quad (\text{A.10})$$

If  $\bar{s}_A < s_A$  or  $q_0 < s_{DM}$ , then, under observable testing, the agent keeps testing at time  $t$  if and only if no signal has arrived by time  $t$  for  $t^* \leq t < T$ , and the DM chooses  $a_1$  if and only if  $\theta = \omega_1$  arrives over time horizon  $(t^*, T]$ . Therefore, the DM's expected payoff under observable testing at time  $t^*$  when no signal has arrived is

$$s_{DM} (1 - \exp(-\lambda^1 (T - t^*))) + (1 - s_{DM}) \frac{s_{DM}}{1 - s_{DM}}. \quad (\text{A.11})$$

Hence, if  $\bar{s}_A < s_A$  or  $q_0 < s_{DM}$ , the DM's expected payoff at time  $t^*$  when no signal has arrived is strictly higher under observable than hidden testing since

$$(\text{A.10}) < (\text{A.11}) \Leftrightarrow \lambda^0 < \lambda^1. \quad (\text{A.12})$$

Otherwise, i.e. if  $s_A \leq \bar{s}_A$  and  $s_{DM} \leq q_0$ , under observable testing, the agent stops at  $t = t^*$  when no signal has arrived. Then  $q_{t^*} = s_{DM}$  and the DM chooses  $a_1$ . Therefore, the DM's expected payoff at time  $t^*$  when no signal has arrived is strictly higher under hidden than observable testing:

$$s_{DM} + (1 - s_{DM}) (1 - \exp(-\lambda^0 (T - t^*))) \frac{s_{DM}}{1 - s_{DM}} > s_{DM}. \quad (\text{A.13})$$

## B Supplementary Appendix

### B.1 Proof of Proposition 5

Among all equilibria under hidden testing ( $p = 0$ ), the agent-preferred equilibrium in Part 2 of Proposition 2 achieves the Pareto-best payoff profile by Propositions 3 and 4. Fixing the strategy profile to be as in Part 2 of Proposition 2, the DM's payoff strictly increases in  $p$ :

$$\begin{aligned} \frac{\partial}{\partial p} & \left[ (1 - q_0) \frac{s_{DM}}{1 - s_{DM}} \left[ (1 - \exp(-\lambda^0(T))) + p \exp(-\lambda^0(T)) \right] \right. \\ & \quad \left. + q_0 \left[ (1 - \exp(-\lambda^1(T))) + (1 - p) \exp(-\lambda^1(T)) \right] \right] > 0 \\ \Leftrightarrow & (1 - q_0) \frac{s_{DM}}{1 - s_{DM}} \exp(-\lambda^0(T)) - q_0 \exp(-\lambda^1(T)) > 0 \\ \Leftrightarrow & \frac{s_{DM}}{1 - s_{DM}} > \frac{q_0 \exp(-\lambda^1(T))}{(1 - q_0) \exp(-\lambda^0(T))} \Leftrightarrow s_{DM} > \underline{q}. \end{aligned}$$

We next show that the strategy profile in Part 2 of Proposition 2 is an equilibrium if and only if  $p \leq p^*$ . Fixing the agent's testing strategy and the DM's strategy, the agent's disclosure strategy is optimal independent of  $p$ , since the agent's disclosure matters only in the continuation game that follows when the DM does not observe  $h^T$  and exogenous disclosure is independent of the agent's findings. Further, given the agent's strategy, the DM's strategy is optimal independent of  $p$ .<sup>43</sup> However, the agent's testing strategy is not necessarily optimal if  $p > 0$ . In particular, if  $p > 0$ , the agent faces a trade-off between testing and stopping if and only if no signal has arrived by time  $t^*$ , where  $t^*$  is given by (4), by the same argument as in the proof of Proposition 1. Suppose no signal has arrived by time  $t^*$ , i.e.  $q_{t^*} = s_{DM}$ . If the agent stops at time  $t^*$ , the DM chooses  $a_1$ . This is because the agent can always disclose  $h^T$  and thereby guarantee that the DM chooses  $a_1$ . Hence, the agent's continuation value from stopping at  $t^*$  is  $V_{t^*}(s_{DM}, t^*) = s_{DM}$ . If the agent keeps testing at time  $t^*$ , the agent will keep testing at all  $t > t^*$  if no signal arrived by time  $t$ , since he does so even if  $p = 1$  by the proof of Proposition 1. Hence, the agent's continuation value from testing at  $t^*$  is

$$\begin{aligned} V_{t^*}(s_{DM}, T) &= (1 - p) \left[ (1 - s_{DM}) (1 - \exp(-\lambda^0(T - t^*))) \frac{s_A}{1 - s_A} + s_{DM} \right] \\ & \quad + p \left[ (1 - s_{DM}) \frac{s_A}{1 - s_A} + s_{DM} (1 - \exp(-\lambda^1(T - t^*))) \right]. \end{aligned}$$

Therefore, the agent continues testing at time  $t^*$  if and only if

$$\begin{aligned} V_{t^*}(s_{DM}, t^*) &\leq V_{t^*}(s_{DM}, T) \\ \Leftrightarrow p &\leq \frac{1 - \exp(-\lambda^0(T - t^*(s_{DM}, q_0)))}{\exp(-\lambda^1(T - t^*(s_{DM}, q_0))) \left( \frac{s_{DM}}{1 - s_{DM}} \right) \left( \frac{1 - s_A}{s_A} \right) - \exp(-\lambda^0(T - t^*(s_{DM}, q_0)))}. \end{aligned} \tag{B.1}$$

<sup>43</sup>That is, the DM reacts in the same way to disclosure of  $h^T$  by the agent or by chance.

Proposition 4 shows that the DM's payoff is strictly higher if the agent tests at time  $t^*$  and  $p = 0$  than if the agent stops at time  $t^*$  even when  $p = 1$ . Therefore, the DM's payoff is maximized if (B.1) is satisfied with equality, i.e. if  $p = p^*$ . At  $p = p^*$ , the agent is indifferent between testing or not at time  $t^*$  when no signal has arrived. Therefore, the agent's payoff is equal to his payoff under observable testing.

If  $s_A = \bar{s}_A$ , then  $p^* = 1$ ,  $p^*$  decreases as  $s_A$  decreases and  $\lim_{s_A \rightarrow 0} p^* = 0$  for any  $s_{DM}, T, q_0$ . Hence,  $p^* \in (0, 1)$ .

## B.2 Proof of Proposition 6

We derive Lemma B.1 on observable testing in subsection B.2.1 and Lemma B.2 on hidden testing in subsection B.2.2. Based on these, we prove Proposition 6 in subsection B.2.3.

### B.2.1 Observable Testing

**Lemma B.1.** *Under observable testing, an equilibrium exists and is unique. The equilibrium strategies are as follows. The DM chooses  $a_1$  if and only if  $s_{DM} \leq q_T$ . There exists some critical time  $\hat{t}(s_A, s_{DM})$  such that*

1. for any  $t < \hat{t}$ , the agent keeps testing at time  $t$ ,
2. for  $t \geq \hat{t}$ , the agent keeps testing at time  $t$  if and only if  $q_t < s_{DM}$ .

The critical time  $\hat{t}$  is defined by  $\hat{t} = \max\{0, T - R\}$  where  $R \in [0, \infty)$  satisfies

$$\log \left( 1 - \frac{\sqrt{R}}{\phi(\sqrt{R}) + \sqrt{R}\Phi(\sqrt{R})} \right) = \log \left( \frac{s_A}{1 - s_A} \right) - \log \left( \frac{s_{DM}}{1 - s_{DM}} \right), \quad (\text{B.2})$$

and  $\phi$  is the density and  $\Phi$  the CDF of a standard Normal distribution. The LHS of (B.2) strictly decreases in  $R$ .

*Proof.* The DM's strategy is optimal by the same arguments as in the proof of Proposition 1. We show that the agent's strategy is a best reply. First, we show that the agent tests at time  $t$  if  $q_t < s_{DM}$  by the following argument. If  $\tau = t$ , then given the DM's strategy,

$$V_t(q_t, t) = (1 - q_t) \frac{s_A}{1 - s_A}. \quad (\text{B.3})$$

If instead  $\tau = t' > t$ , then given the DM's strategy,

$$V_t(q_t, t') = (1 - q_t) \frac{s_A}{1 - s_A} \Pr(q_{t'} < s_{DM} | q_t, \tau = t', \omega_0) + q_t \Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_1).$$

The agent keeps testing at any time  $t$  if  $q_t < s_{DM}$  since

$$V_t(q_t, t') - V_t(q_t, t) > 0 \Leftrightarrow \frac{\Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_1)}{\Pr(q_{t'} \geq s_{DM} | q_t, \tau = t', \omega_0)} \frac{q_t}{1 - q_t} > \frac{s_A}{1 - s_A} \quad (\text{B.4})$$

$$\Leftrightarrow \frac{\Pr(\omega_1 | q_t, \tau = t', q_{t'} \geq s_{DM})}{\Pr(\omega_0 | q_t, \tau = t', q_{t'} \geq s_{DM})} > \frac{s_A}{1 - s_A}, \quad (\text{B.5})$$

where the second step follows by Bayes' rule and the inequality is satisfied for any  $t' > t$  since

$$\Pr(\omega_1 | q_t, \tau = t', q_{t'} \geq s_{DM}) > \Pr(\omega_1 | q_t, \tau = t', q_{t'} = s_{DM}) = s_{DM} \geq s_A, \quad (\text{B.6})$$

and  $\frac{x}{1-x} > \frac{s_A}{1-s_A}$  if and only if  $x > s_A$  for any  $x \in (0, 1)$ .

Next, consider time  $t$  with  $q_t > s_{DM}$  and suppose the agent tests until the first time  $\tau$  that  $q = s_{DM}$ . Therefore,  $q_T \geq s_{DM}$  and  $V_t(q_t, \tau) = V_t(q_t, t)$ , i.e. the agent cannot be strictly worse off by testing until time  $\tau$ .

Finally, consider time  $t$  with  $q_t = s_{DM}$ . We start by showing that, for a given  $s_A$ , there exists a critical level of the time remaining  $R(s_{DM}, s_A) \in [0, \infty)$  such that the agent stops if and only if  $T - t \leq R(s_{DM}, s_A)$ . First, note that the continuation value of stopping at time  $t$ ,  $V_t(s_{DM}, t)$ , is independent of the time remaining. Second, the continuation value from using the optimal stopping time, denoted by  $V_t(s_{DM}, \tau^*)$ , must satisfy  $V_t(s_{DM}, \tau^*) \geq V_t(s_{DM}, t)$  by optimality. Third,  $V_t(s_{DM}, \tau^*)$  cannot strictly decrease as the time remaining increases because the agent's set of possible stopping times increases.<sup>44</sup> The above implies that, if there exists  $t$  such that  $V_t(s_{DM}, \tau^*) = V_t(s_{DM}, t)$ , then for any  $t' > t$ ,  $V_{t'}(s_{DM}, \tau^*) = V_{t'}(s_{DM}, t)$ . To show that there exists  $t$  such that  $V_t(s_{DM}, \tau^*) = V_t(s_{DM}, t)$ , compare the continuation value of stopping at  $t$  with the continuation value of testing until the deadline  $T$ :

$$\begin{aligned} V_t(s_{DM}, t) \geq V_t(s_{DM}, T) &\Leftrightarrow \frac{s_{DM}}{1 - s_{DM}} \frac{\Pr(q_T < s_{DM} | q_t = s_{DM}, \tau = T, \omega_1)}{\Pr(q_T < s_{DM} | q_t = s_{DM}, \tau = T, \omega_0)} \geq \frac{s_A}{1 - s_A} \\ &\Leftrightarrow \frac{\Phi(-\sqrt{T-t})}{\Phi(\sqrt{T-t})} \geq \frac{s_A}{1 - s_A} \frac{1 - s_{DM}}{s_{DM}}. \end{aligned} \quad (\text{B.7})$$

As  $t \rightarrow T$ , the LHS of (B.7) tends to 1 and the inequality is satisfied for any  $s_A \leq s_{DM}$ , i.e. stopping yields a strictly higher continuation value sufficiently close to the deadline  $T$ .

The next step is to show that the critical level of the time remaining  $R$  satisfies (B.2). First, let  $R' \in (0, T)$  be an amount of time remaining such that  $V_{T-R'}(s_{DM}, T - R') \geq V_{T-R'}(s_{DM}, T)$ , implying that the agent stops at  $q = s_{DM}$  if  $T - t \leq R'$ . Second, fix some  $\epsilon \in (0, R')$ . Consider  $t$  with  $T - t \geq R'$  and  $q_t = s_{DM}$ . Consider the following stopping rule  $\tau_\epsilon$ : the agent tests at any time in  $[t, t + \epsilon)$  and then the agent stops at the first time that  $q \geq s_{DM}$ . Take  $T$  to be sufficiently large. Then by the previous argument, as  $T - t$  increases above  $R'$ , there exists a critical level of the time remaining  $R_\epsilon$  at which the agent is indifferent between stopping at  $T - R_\epsilon$  and following stopping

<sup>44</sup>To see why, suppose  $q_t = s_{DM}$  and suppose the deadline  $T$  increases to  $T' > T$ . If the deadline is at time  $T$  and the agent optimally stops at some time  $\tau \leq T$ , then he can achieve the same payoff by stopping at time  $\tau$  when the deadline is  $T' > T$ . However, if the deadline is at  $T'$ , the agent also has the option to continue testing at  $T$  and, therefore, his payoff must be weakly higher.

rule  $\tau_\epsilon$ ,<sup>45</sup> i.e.

$$\begin{aligned}
& V_{T-R_\epsilon}(s_{DM}, \tau_\epsilon) = V_{T-R_\epsilon}(s_{DM}, T - R_\epsilon) \\
& \Leftrightarrow \frac{\Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_1\right)}{\Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega_0\right)} = \frac{s_A}{1-s_A} \frac{1-s_{DM}}{s_{DM}}, \tag{B.8}
\end{aligned}$$

where

$$\begin{aligned}
& \Pr\left(\max_{k \in [T-R_\epsilon+\epsilon, T]} q_k < s_{DM} \mid q_{T-R_\epsilon} = s_{DM}, \omega\right) = \\
& \int_{-\infty}^{s_{DM}} \Pr\left(\Sigma_\epsilon = \frac{1}{2}(x - s_{DM}) \mid \Sigma_0 = 0, \omega\right) \Pr\left(\max_{k \in [0, R_\epsilon-\epsilon]} \Sigma_k < \frac{1}{2}(s_{DM} - x) \mid \Sigma_0 = 0, \omega\right) dx, \tag{B.9}
\end{aligned}$$

and  $\Sigma_t \equiv \frac{1}{2}Q_t = \mu_\omega dt + dW_t$  and  $S_{DM} \equiv \log\left(\frac{s_{DM}}{1-s_{DM}}\right)$ .

The next step is to show that, in the limit as  $\epsilon \rightarrow 0$ , (B.8) is equivalent to (B.2). Define

$$m_\epsilon(x; \omega) \equiv Pr\left(\max_{k \in [0, R_\epsilon-\epsilon]} \Sigma_k < \frac{1}{2}(s_{DM} - x) \mid \Sigma_0 = 0, \omega\right). \tag{B.10}$$

The Taylor expansion of  $m_\epsilon(x; \omega)$  about  $x = S_{DM}$  is given by

$$m'_\epsilon(S_{DM}; \omega)(x - S_{DM}) + r_{1,\epsilon}(x; \omega). \tag{B.11}$$

Substituting (B.11) into (B.9):

$$\begin{aligned}
& \int_{-\infty}^{s_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - s_{DM}) - \mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) [m'_\epsilon(S_{DM}; \omega)(x - S_{DM}) + r_{1,\epsilon}(x; \omega)] dx \\
& = m'_\epsilon(S_{DM}; \omega) \int_{-\infty}^{s_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - s_{DM}) - \mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) (x - S_{DM}) dx \\
& + \int_{-\infty}^{s_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x - s_{DM}) - \mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) r_{1,\epsilon}(x; \omega) dx. \tag{B.12}
\end{aligned}$$

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<sup>45</sup>The final action choice is affected by following stopping time  $\tau_\epsilon$  instead of stopping at time  $T - R_\epsilon$  if the belief drops below the DM's standard within  $[T - R_\epsilon, T - R_\epsilon + \epsilon)$  and never exceeds the DM's standard within  $[T - R_\epsilon + \epsilon, T]$ .

Therefore, taking the limit of the LHS of (B.8) as  $\epsilon \rightarrow 0$  yields:

$$\lim_{\epsilon \rightarrow 0} \frac{\Pr \left( \max_{k \in [T - \bar{R}_\epsilon + \epsilon, T]} q_k < s_{DM} \mid q_{T - \bar{R}_\epsilon} = s_{DM}, \omega_1 \right)}{\Pr \left( \max_{k \in [T - \bar{R}_\epsilon + \epsilon, T]} q_k < s_{DM} \mid q_{T - \bar{R}_\epsilon} = s_{DM}, \omega_0 \right)} = \lim_{\epsilon \rightarrow 0} \frac{m'_\epsilon(S_{DM}; \omega_1) + n_\epsilon(S_{DM}; \omega_1)}{m'_\epsilon(S_{DM}; \omega_0) + n_\epsilon(S_{DM}; \omega_0)}, \quad (\text{B.13})$$

where

$$n_\epsilon(S_{DM}; \omega) = \frac{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp \left( -\frac{1}{2} \left( \frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}} \right)^2 \right) r_{1,\epsilon}(x; \omega) dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp \left( -\frac{1}{2} \left( \frac{\frac{1}{2}(x - S_{DM}) - \mu_\omega \epsilon}{\sqrt{\epsilon}} \right)^2 \right) (x - S_{DM}) dx}. \quad (\text{B.14})$$

To calculate  $m'_\epsilon(S_{DM}; \omega)$ , we use the following standard result repeated here for convenience: the CDF of  $\max_{t \in [0, T]} \tilde{W}_t$  of a drifted Brownian motion  $\tilde{W}_t = \gamma t + W_t$  over  $t \in [0, T]$  with  $\tilde{W}_0 = 0$  is given by:

$$\Pr \left( \max_{t \in [0, T]} \tilde{W}_t < a \right) = \Phi \left( \frac{a - \gamma T}{\sqrt{T}} \right) - \exp(2\gamma a) \Phi \left( \frac{-a - \gamma T}{\sqrt{T}} \right) \quad (\text{B.15})$$

for  $a \geq 0$ . Hence,

$$m_\epsilon(x; \omega) = \Phi \left( \frac{\frac{1}{2}(S_{DM} - x) - \mu_\omega (R_\epsilon - \epsilon)}{\sqrt{R_\epsilon - \epsilon}} \right) - \exp(\mu_\omega (S_{DM} - x)) \Phi \left( \frac{-\frac{1}{2}(S_{DM} - x) - \mu_\omega (R_\epsilon - \epsilon)}{\sqrt{R_\epsilon - \epsilon}} \right), \quad (\text{B.16})$$

$$m'_\epsilon(S_{DM}; \omega) = -\frac{1}{\sqrt{R_\epsilon - \epsilon}} \phi \left( -\mu_\omega \sqrt{R_\epsilon - \epsilon} \right) + \mu_\omega \Phi \left( -\mu_\omega \sqrt{R_\epsilon - \epsilon} \right). \quad (\text{B.17})$$

Suppose  $\lim_{\epsilon \rightarrow 0} n_\epsilon(S_{DM}; \omega) = 0$ . Then (B.13) and (B.17) yield

$$\lim_{\epsilon \rightarrow 0} \frac{\Pr \left( \max_{k \in [T - \bar{R}_\epsilon + \epsilon, T]} q_k < s_{DM} \mid q_{T - \bar{R}_\epsilon} = s_{DM}, \omega_1 \right)}{\Pr \left( \max_{k \in [T - \bar{R}_\epsilon + \epsilon, T]} q_k < s_{DM} \mid q_{T - \bar{R}_\epsilon} = s_{DM}, \omega_0 \right)} = 1 - \frac{\sqrt{\bar{R}}}{\phi(\sqrt{\bar{R}}) + \sqrt{\bar{R}}\Phi(\sqrt{\bar{R}})}. \quad (\text{B.18})$$

Therefore, in the limit as  $\epsilon \rightarrow 0$ , (B.8) is equivalent to (B.2). Hence, the critical time  $t$  must satisfy  $\hat{t} = \max\{0, T - R\}$  where  $R$  is defined by (B.2). See also Figure 3.

The LHS of (B.2) strictly decreases in  $R$ :

$$\frac{\partial}{\partial R} \left[ \log \left( 1 - \frac{\sqrt{\bar{R}}}{\phi(\sqrt{\bar{R}}) + \sqrt{\bar{R}}\Phi(\sqrt{\bar{R}})} \right) \right] = \frac{\partial}{\partial R} \left[ 1 - \frac{1}{g(R)} \right] = \frac{1}{1 - \frac{1}{g(R)}} \left[ \frac{g'(R)}{g(R)^2} \right] < 0, \quad (\text{B.19})$$

where

$$g(R) = \frac{1}{\sqrt{R}}\phi(\sqrt{R}) + \Phi(\sqrt{R}), \quad (\text{B.20})$$

$$g'(R) = -\frac{\exp(-\frac{1}{2}R)}{2\sqrt{2\pi}R^{\frac{3}{2}}} < 0, \quad (\text{B.21})$$

and  $g(R) \geq 1$  for all  $R \geq 0$  as  $\lim_{R \rightarrow \infty} g(R) = 1$ .

The rest of the proof shows that  $\lim_{\epsilon \rightarrow 0} n_\epsilon(S_{DM}; \omega) = 0$ . The remainder term in (B.11):

$$r_{1,\epsilon}(x; \omega) = m_\epsilon''(S_{DM}; \omega)(x - S_{DM}) + r_{2,\epsilon}(x; \omega), \quad (\text{B.22})$$

Rewrite (B.14) as

$$\begin{aligned} & \frac{m_\epsilon''(S_{DM}; \omega) \int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) (x - S_{DM})^2 dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) (x - S_{DM}) dx} \\ & + \frac{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) r_{2,\epsilon}(x; \omega) dx}{\int_{-\infty}^{S_{DM}} \frac{1}{\sqrt{2\pi\epsilon}} \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{2}(x-S_{DM})-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right) (x - S_{DM}) dx}. \end{aligned} \quad (\text{B.23})$$

The first term of (B.23) can be written as

$$m_\epsilon''(S_{DM}; \omega) \frac{\int_{-\infty}^0 \frac{1}{\sqrt{2\pi\epsilon}} \exp\left\{-\frac{1}{2}\left(\frac{y-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right\} y^2 dy}{\int_{-\infty}^0 \frac{1}{\sqrt{2\pi\epsilon}} \exp\left\{-\frac{1}{2}\left(\frac{y-\mu_\omega\epsilon}{\sqrt{\epsilon}}\right)^2\right\} y dy} = m_\epsilon''(S_{DM}; \omega) \frac{E(Y^2|Y < 0)}{E(Y|Y < 0)}, \quad (\text{B.24})$$

where  $Y \sim N(\mu_\omega\epsilon, \sqrt{\epsilon})$ . Then for  $Z \sim N(0, 1)$ ,

$$E(Y|Y < 0) = E(\sqrt{\epsilon}Z + \mu_\omega\epsilon | Z < -\mu_\omega\sqrt{\epsilon}) = \sqrt{\epsilon} \left[ \sqrt{\epsilon}\mu_\omega - \frac{\phi(-\mu_\omega\sqrt{\epsilon})}{\Phi(-\mu_\omega\sqrt{\epsilon})} \right] \quad (\text{B.25})$$

$$E(Y^2|Y < 0) = E\left(\left(\sqrt{\epsilon}Z + \mu_\omega\epsilon\right)^2 | Z < -\mu_\omega\sqrt{\epsilon}\right) = \epsilon + \epsilon^2 - \mu_\omega\epsilon\sqrt{\epsilon} \frac{\phi(-\mu_\omega\sqrt{\epsilon})}{\Phi(-\mu_\omega\sqrt{\epsilon})} \quad (\text{B.26})$$

Substituting (B.25) and (B.26) into (B.24) and taking limits yields

$$\lim_{\epsilon \rightarrow 0} \frac{E(Y^2|Y < 0)}{E(Y|Y < 0)} m_\epsilon''(S_{DM}; \omega) = m_\epsilon''(S_{DM}; \omega) \lim_{\epsilon \rightarrow 0} \frac{\sqrt{\epsilon} + \sqrt{\epsilon}\epsilon - \mu_\omega\epsilon \frac{\phi(-\mu_\omega\sqrt{\epsilon})}{\Phi(-\mu_\omega\sqrt{\epsilon})}}{\left[ \sqrt{\epsilon}\mu_\omega - \frac{\phi(-\mu_\omega\sqrt{\epsilon})}{\Phi(-\mu_\omega\sqrt{\epsilon})} \right]} = \frac{0}{-\sqrt{\frac{2}{\pi}}} = 0.$$

The same reasoning can be applied reiteratively to the second term of (B.23).  $\square$

### B.2.2 Hidden testing

**Lemma B.2.** *Suppose testing is hidden. There exists a critical level of the DM's standard  $\widehat{s}_{DM}(q_0, T, s_A)$ , where  $\widehat{s}_{DM} \in (q_0, 1)$  is defined by*

$$\log\left(\frac{\widehat{s}_{DM}}{1 - \widehat{s}_{DM}}\right) = Q_0 + \log\left(\frac{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) - T}{\sqrt{T}}\right)}{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) + T}{\sqrt{T}}\right)}\right), \quad (\text{B.27})$$

such that if  $s_{DM} \leq \widehat{s}_{DM}$ , an equilibrium exists, and the following are features of an agent-preferred equilibrium,

- the agent tests until  $t = T$ , and discloses  $h^T$  if and only if he prefers  $a_0$  given  $h^T$ ,
- the DM chooses  $a_0$  if and only if the agent discloses  $h^T$  and the DM prefers  $a_0$  given  $h^T$ .

*Proof.* Given that the agent sends  $m = \emptyset$  if and only if  $q_T \geq s_A$ , the DM's consistent beliefs following  $m = \emptyset$  satisfy

$$\frac{\Pr(\omega_1 | m = \emptyset)}{\Pr(\omega_0 | m = \emptyset)} = \frac{q_0}{1 - q_0} \frac{\Pr(q_T \geq s_A | q_0, \tau = T, \omega_1)}{\Pr(q_T \geq s_A | q_0, \tau = T, \omega_0)} = \frac{q_0}{1 - q_0} \frac{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) - T}{\sqrt{T}}\right)}{1 - \Phi\left(\frac{\frac{1}{2}\left(\log\left(\frac{s_A}{1-s_A}\right) - Q_0\right) + T}{\sqrt{T}}\right)}, \quad (\text{B.28})$$

hence,  $\Pr(\omega_1 | m = \emptyset) = \widehat{s}_{DM}$  by (B.27). Since  $s_{DM} \leq \widehat{s}_{DM}$ , the DM optimally chooses  $a_1$  following  $m = \emptyset$ . In addition, the agent has no reason to deviate from his strategy because he achieves his first-best payoff. This also implies that the equilibrium is agent-preferred.  $\square$

### B.2.3 Welfare comparison

We now prove Proposition 6. Define  $\widehat{s}_A(s_{DM}, T)$  such that (B.2) is satisfied when  $s_A = \widehat{s}_A$  and  $R = T$ .

The agent's equilibrium payoff under observable testing is strictly less than his first-best payoff as the agent stops before time  $T$  with positive probability, while in the agent-preferred equilibrium in Lemma B.2, the agent obtains his first-best payoff. Hence, the agent must have a strictly higher payoff in the agent-preferred equilibrium under hidden testing than in the equilibrium under observable testing.

We will distinguish between two cases to compare the DM's payoff under hidden and observable testing: i)  $s_{DM} \leq q_0$  and  $s_A \leq \widehat{s}_A$  and ii)  $q_0 < s_{DM} \leq \widehat{s}_{DM}$  and  $s_A \leq \widehat{s}_A$ .<sup>46</sup>

In both case i) and case ii), by Lemma B.2, in the agent-preferred equilibrium under hidden testing,  $a_1$  is chosen with probability  $\Pr(q_T \geq s_A | q_0, \tau = T)$ , i.e. with the probability that  $q_T \geq s_A$  given the prior is  $q_0$  and the agent tests until time  $T$ .

<sup>46</sup>Note that  $q_0 < \widehat{s}_{DM}$  by Lemma B.2.

Consider case i), i.e.  $s_{DM} \leq q_0$  and  $s_A \leq \hat{s}_A$ . By Lemma B.1,  $\hat{t} = 0$  and, hence, under observable testing  $a_1$  is chosen with probability 1. Then the DM has a strictly higher payoff under hidden than observable testing since

$$(1 - q_0) \frac{s_{DM}}{1 - s_{DM}} \Pr(q_T < s_A | q_0, \tau = T, \omega_0) + q_0 \Pr(q_T \geq s_A | q_0, \tau = T, \omega_1) > q_0$$

$$\Leftrightarrow \frac{s_{DM}}{1 - s_{DM}} > \frac{\Pr(\omega_1 | q_T < s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T < s_A, q_0, \tau = T)}$$

which holds since

$$\frac{s_{DM}}{1 - s_{DM}} \geq \frac{s_A}{1 - s_A} = \frac{\Pr(\omega_1 | q_T = s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T = s_A, q_0, \tau = T)} > \frac{\Pr(\omega_1 | q_T < s_A, q_0, \tau = T)}{\Pr(\omega_0 | q_T < s_A, q_0, \tau = T)}.$$

The set of  $s_{DM}$  for which  $s_{DM} \leq q_0$  is non-empty. In addition, the set of  $s_A$  for which  $s_A \leq \hat{s}_A$  is non-empty since  $\hat{s}_A(s_{DM}, T) > 0$  for any  $s_{DM}$  and finite  $T$  by Lemma B.1. Hence, for any  $q_0$  and  $T$  the region for which  $s_{DM} \leq q_0$  and  $s_A \leq \hat{s}_A$  is non-empty.

Next, consider case ii), i.e.  $s_A \leq \hat{s}_A$  and  $q_0 < s_{DM} \leq \hat{s}_{DM}$ , or equivalently,

$$Q_0 < \log\left(\frac{s_{DM}}{1 - s_{DM}}\right) \leq Q_0 + \log\left(\frac{1 - \Phi\left(\frac{\frac{1}{2}(\log(\frac{s_A}{1 - s_A}) - Q_0) - T}{\sqrt{T}}\right)}{1 - \Phi\left(\frac{\frac{1}{2}(\log(\frac{s_A}{1 - s_A}) - Q_0) + T}{\sqrt{T}}\right)}\right). \quad (\text{B.29})$$

By Lemma B.1,  $a_1$  is chosen with probability  $\Pr(\max_{t \in [0, T]} q_t \geq s_{DM} | q_0, \tau = T)$ . Then the DM has a strictly higher payoff under hidden than observable testing if and only if

$$\Pr(q_T < s_A | q_0, \tau = T, \omega_0) + \frac{q_0}{1 - q_0} \frac{1 - s_{DM}}{s_{DM}} \Pr(q_T \geq s_A | q_0, \tau = T, \omega_1) >$$

$$\Pr\left(\max_{t \in [0, T]} q_t < s_{DM} | q_0, \tau = T, \omega_0\right) + \frac{q_0}{1 - q_0} \frac{1 - s_{DM}}{s_{DM}} \Pr\left(\max_{t \in [0, T]} q_t \geq s_{DM} | q_0, \tau = T, \omega_1\right). \quad (\text{B.30})$$

The LHS of (B.30) can be written as

$$\Phi\left(\frac{\frac{1}{2}(S_A - Q_0) + T}{\sqrt{T}}\right) - \exp(Q_0 - S_{DM}) \Phi\left(\frac{\frac{1}{2}(S_A - Q_0) - T}{\sqrt{T}}\right), \quad (\text{B.31})$$

where  $S_i \equiv \log\left(\frac{s_i}{1 - s_i}\right)$  for  $i \in \{A, DM\}$ , and the RHS can be written as

$$\Phi\left(\frac{\frac{1}{2}(S_{DM} - Q_0) + T}{\sqrt{T}}\right) - \exp(Q_0 - S_{DM}) \Phi\left(\frac{-\frac{1}{2}(S_{DM} - Q_0) + T}{\sqrt{T}}\right)$$

$$- \exp(Q_0 - S_{DM}) \left[ \Phi\left(\frac{\frac{1}{2}(S_{DM} - Q_0) - T}{\sqrt{T}}\right) - \exp(S_{DM} - Q_0) \Phi\left(\frac{-\frac{1}{2}(S_{DM} - Q_0) - T}{\sqrt{T}}\right) \right]$$

$$= 1 - \exp(Q_0 - S_{DM}), \quad (\text{B.32})$$

using (B.15). Substituting (B.31) and (B.32) into (B.30), we can see that (B.30) is implied by the second inequality of (B.29).