B Supplementary Appendix

Contents

B.1 Lemma 11: Advisor-preferred Equilibria

To show that the equilibria are sequential equilibria, I consider a completely mixed strategy for the advisor as a function of some $\varepsilon \in (0,1)$, denoted by $\left(\underline{\sigma}_A^{T\varepsilon}, \underline{\sigma}_A^{M\varepsilon}\right)$, which is constructed as follows:

$$\underline{\sigma}_A^{T\varepsilon}(h_0) = \begin{cases} 1-\varepsilon & \text{if } \underline{\sigma}_A^T(h_0) = 1 \\ \varepsilon & \text{if } \underline{\sigma}_A^T(h_0) = 0, \end{cases} \tag{100}$$

$$\underline{\sigma}_A^{T\varepsilon}(h_1) = \begin{cases} 1-\varepsilon^2 & \text{if } \underline{\sigma}_A^T(h_1) = 1 \\ \varepsilon^2 & \text{if } \underline{\sigma}_A^T(h_1) = 0, \end{cases} \tag{101}$$

and for any $\widetilde{h}$ and associated set of feasible messages $M\left(\widetilde{h}\right)$, $\underline{\sigma}_A^M(m)$ is sent with probability $1 - \left(|M\left(\widetilde{h}\right)| - 1\right)\varepsilon^4$ and any other message $m \in M\left(\widetilde{h}\right)$ is sent with probability ε^4, where $|M\left(\widetilde{h}\right)|$ denotes the total number of feasible messages given $\widetilde{h}$. Clearly, as $\varepsilon \to 0$, $\left(\underline{\sigma}_A^{T\varepsilon}, \underline{\sigma}_A^{M\varepsilon}\right) \to \left(\underline{\sigma}_A^T, \underline{\sigma}_A^M\right)$. In addition, I show that the beliefs derived from the advisor's mixed strategy profile using Bayes' rule, denoted by $\underline{\mu}^\varepsilon(m)$, satisfy

$$\frac{\underline{\mu}^\varepsilon(m)}{1-\underline{\mu}^\varepsilon(m)} \to \frac{\underline{\mu}(m)}{1-\underline{\mu}(m)}. \tag{102}$$

Note that this must trivially be the case for any message $m \in \{\{+,+\},\{+,-\},\{-,-\}\}$ due to verifiable disclose.

Region 1a and Region 2a), i.e. $\lambda_A < l_{(+,-)}$ and $\lambda_{DM} < l_{\neg(-,-)}$. In the limit as $\varepsilon \to 0$, $m = \{-\}$ is mostly likely observed following $h_2 \in \{(-,-),(-,+)\}$. $m = \{-\}$ always requires a deviation from $\underline{\sigma}_A^M$. In addition, each $h_2 \in \{(-,-),(-,+)\}$ occurs on path, whereas any other history for which $m = \{-\}$ is feasible is reached only after a deviation from $\underline{\sigma}_A^T$.

Region 1b). In the limit as $\varepsilon \to 0$, $m = \{+\}$ is most likely observed following $h_2 = (\emptyset,+)$. $h_2 = (\emptyset,+)$ occurs with a probability of order ε, since it results in $m = \{+\}$ given $\underline{\sigma}_A^M$

and arises after a single deviation from $\underline{\sigma}_A^T$ in period 1. Any other history that results in $m = \{+\}$ occurs with a probability of at least order ε^2. In the limit as $\varepsilon \to 0$, $m = \emptyset$ is mostly likely observed following $h_2 = (\emptyset, \emptyset)$. This history occurs with a probability of order ε^3, since it results in $m = \emptyset$ given $\underline{\sigma}_A^M$ and arises after two deviations from $\underline{\sigma}_A^T$. Any other history resulting in $m = \emptyset$ occurs with a probability of at least order ε^4, because a deviation from $\underline{\sigma}_A^M$ is necessary for it to arise.

Region 1c). In the limit as $\varepsilon \to 0$, $m = \{+\}$ is most likely observed following $h_2 \in \{(+,+), (+,-)\}$, since each of these histories occurs given $\underline{\sigma}_A^T$ and results in $m = \{+\}$ if and only if there is a single deviation from $\underline{\sigma}_A^M$, which occurs with a probability of order ε^4. For any other history to result in $m = \{+\}$ both a deviation from $\underline{\sigma}_A^M$ and a deviation from $\underline{\sigma}_A^T$ is necessary, which occurs with a probability of at least order ε^5. In the limit as $\varepsilon \to 0$, $m = \emptyset$ is most likely observed following $h_2 = (\emptyset, \emptyset)$, since this history results in $m = \emptyset$ given $\underline{\sigma}_A^M$ and requires a single deviation from $\underline{\sigma}_A^T$ in the first period, which occurs with a probability of order ε. Any other history which results in $m = \emptyset$ requires at least two deviations from $\underline{\sigma}_A^T$, which occur with a probability of order ε^3.

Region 2b). In the limit as $\varepsilon \to 0$, $m = \{-\}$ is most likely observed following $h_2 \in \{(-, \emptyset), (+,-)\}$. Each of these histories occurs on path and results in $m = \{-\}$ with a probability of order ε^4, since a deviation from $\underline{\sigma}_A^M$ is required. For any other history to results in $m = \{-\}$, a deviation from both $\underline{\sigma}_A^T$ and $\underline{\sigma}_A^M$ is required.

Region 2c). Apply the same reasoning as in in Region 1b.

Region 2d). Apply the same reasoning as in in Region 1c.

Region 3. Apply the same reasoning as in Region 2.

Region 4. In the limit as $\varepsilon \to 0$, $m = \{-\}$ ($m = \{+\}$) is most likely observed following $h_2 = (-, \emptyset)$ ($h_2 = (+, \emptyset)$). Since this history arises after a single deviation from $\underline{\sigma}_A^T$ in period 1 and no deviation from $\underline{\sigma}_A^M$, it occurs with a probability of order ε. For any other history to result in m at least two deviations from $\underline{\sigma}_A^T$ or a deviation from $\underline{\sigma}_A^M$ are required, which means it occurs with a probability of at least order ε^3.

Finally, if $\lambda_A < l_{(+,+)}$, the equilibrium is a sequential equilibrium by the same reasoning as in Region 4.

B.2 Lemma 12: DM-preferred Equilibria

Proof: Part 1: To show that $(\overline{\sigma}, \overline{\mu})$ is a sequential equilibrium, consider a completely mixed strategy for the advisor as a function of some $\varepsilon \in (0,1)$, denoted by $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right)$, which is constructed as follows: for any history of outcomes h,

$$\overline{\sigma}_A^{T\varepsilon}(h) = \begin{cases} 1-\varepsilon^3 & \text{if } \overline{\sigma}_A^T(h) = 1 \\ \varepsilon^3 & \text{if } \overline{\sigma}_A^T(h) = 0, \end{cases} \tag{103}$$

and if $\widetilde{h} = \{+,-\}$ then $m = \emptyset$ is sent with probability ε, $m \in \{\{+\},\{+,-\}\}$ is sent with probability ε^3 and $m = \{-\}$ is sent with probability $1-\varepsilon-2\varepsilon^3$. Otherwise, for any $\widetilde{h}$ and set of feasible messages $M\left(\widetilde{h}\right)$, the message specified by $\overline{\sigma}_A^M$ is sent with probability $1-\left(|M\left(\widetilde{h}\right)|-1\right)\varepsilon^3$ and any other feasible message $m \in M\left(\widetilde{h}\right)$ is sent with probability ε^3, where $|M\left(\widetilde{h}\right)|$ denotes the total number of feasible messages given $\widetilde{h}$. As $\varepsilon \to 0$, clearly $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right) \to \left(\overline{\sigma}_A^T, \overline{\sigma}_A^M\right)$. In addition, the beliefs derived from the advisor's mixed strategy profile using Bayes' rule, denoted by $\overline{\mu}^\varepsilon(m)$, satisfy (102). In the limit as $\varepsilon \to 0$, $m = \emptyset$ is most likely observed following $h_2 = (-,+)$. This history occurs on path and results in $m = \emptyset$ after a deviation from $\overline{\sigma}_A^M$. Hence, this history occurs and results in $m = \emptyset$ with a probability of order ε. All other histories occur and result in $m = \emptyset$ with a probability of at least order ε^3.

Part 2. Consider a completely mixed strategy for the advisor as a function of some $\varepsilon \in (0,1)$, denoted by $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right)$, which is constructed as follows: for any history of outcomes h,

$$\overline{\sigma}_A^{T\varepsilon}(h) = \begin{cases} 1-\varepsilon & \text{if } \overline{\sigma}_A^T(h) = 1 \\ \varepsilon & \text{if } \overline{\sigma}_A^T(h) = 0, \end{cases} \tag{104}$$

and for any $\widetilde{h}$ and associated set of feasible messages $M\left(\widetilde{h}\right)$, the message specified by $\overline{\sigma}_A^M$ is sent with probability $1-\left(|M\left(\widetilde{h}\right)|-1\right)\varepsilon^3$ and any other feasible message $m \in M\left(\widetilde{h}\right)$ is sent with probability ε^3, where $|M\left(\widetilde{h}\right)|$ denotes the total number of feasible messages given $\widetilde{h}$. As $\varepsilon \to 0$, clearly $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right) \to \left(\overline{\sigma}_A^T, \overline{\sigma}_A^M\right)$. Furthermore, for any m, as $\varepsilon \to 0$ (102) holds for the following reasons: In the limit as $\varepsilon \to 0$, $m = \{+\}$ is mostly likely observed following $h_2 \in \{(+),(-,+)\}$. Each of these histories occurs and result in $m = \{+\}$ with a probability of order ε^3, since they occur on path and $m = \{+\}$ is sent due to a deviation from $\overline{\sigma}_A^M$. All other histories occur and result in $m = \{+\}$ with a probability of at least order ε^4 because they require both a deviation from $\overline{\sigma}_A^T$ and $\overline{\sigma}_A^M$.

Part 3: The equilibrium is a sequential equilibrium by the same reasoning as for Part 2.

Part 4: Consider a completely mixed strategy constructed as in Part 2. As $\varepsilon \to 0$, $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right) \to \left(\overline{\sigma}_A^T, \overline{\sigma}_A^M\right)$. Furthermore, for any m, as $\varepsilon \to 0$ (102) holds for the following reasons: In the limit as $\varepsilon \to 0$, $m = \emptyset$ is most likely observed following $h_2 \in \{(\emptyset,\emptyset)\}$.

This history occurs and is followed by $m = \emptyset$ with a probability of order ε^2 since it requires two deviations from $\overline{\sigma}_A^T$. All other histories occur and are followed by $m = \emptyset$ with probability of at least order ε^3 because they require a deviation from $\overline{\sigma}_A^M$.

Part 5. Consider a completely mixed strategy constructed as in Part 2. As $\varepsilon \to 0$, $\left(\overline{\sigma}_A^{T\varepsilon}, \overline{\sigma}_A^{M\varepsilon}\right) \to \left(\overline{\sigma}_A^T, \overline{\sigma}_A^M\right)$. Furthermore, for any m, as $\varepsilon \to 0$ (102) holds for the following reasons: In the limit as $\varepsilon \to 0$, $m = \{+\}$ is most likely observed following $h_2 \in \{(+,+),(+,-)\}$, since each of these histories occurs and results in $m = \{+\}$ with a probability of order ε^3 (due to a deviation from $\overline{\sigma}_A^M$) whereas all other histories occur and result in $m = \{+\}$ with probability of at least order ε^4, because they require both a deviation from $\overline{\sigma}_A^M$ and $\overline{\sigma}_A^T$.

B.3 Lemma 7 [Hidden Testing for $N > 2$]

Proof Claim 1d: Consider the following completely mixed strategy for the advisor with $\varepsilon \in (0, 1)$:

$$\underline{\sigma}_A^{\varepsilon T}(h_n) = \begin{cases} \varepsilon^N & \text{if } \nu_n^+ \geq \overline{\nu}_+ \text{ or if } \nu_n^- \geq \overline{\nu}_-, \\ 1 - \varepsilon^N & \text{otherwise,} \end{cases} \tag{105}$$

and $\underline{\sigma}_A^{\varepsilon M}$ is as follows. If $x_N \leq r$, fails to report each negative outcome in $\underline{\sigma}_A^M$ independently with probability ε and reports any strictly positive number of positive outcomes with probability ε. If $x_N \geq r$ the advisor fails to report each positive outcome in $\underline{\sigma}_A^M$ independently with probability ε^N and reports any strictly positive number of negative outcome with probability ε.

Clearly, $\left(\underline{\sigma}_A^{T\varepsilon}, \underline{\sigma}_A^{M\varepsilon}\right) \to \left(\underline{\sigma}_A^T, \underline{\sigma}_A^M\right)$ as $\varepsilon \to 0$. I show that the DM's beliefs also converge. If $\varepsilon = 0$, $m = (\overline{\nu}_+, 0)$ occurs following a history in which the advisor stops testing as soon as $\nu_n^+ = \overline{\nu}_+$ and reports all positive outcomes. Therefore, if $\varepsilon = 0$, $m = (\overline{\nu}_+, 0)$ arises if and only if at least $\overline{\nu}_+$ outcomes in N tests are positive. If $\varepsilon = 0$ then $m = (0, \overline{\nu}_-)$ occurs following a history in which when the advisor stops testing as soon as $\nu_n^- = \overline{\nu}_-$ and reports all negative outcomes. Therefore, if $\varepsilon = 0$, $m = (0, \overline{\nu}_-)$ arises if and only if at least $\overline{\nu}_-$ outcomes in N tests are negative.

In the limit as $\varepsilon \to 0$, any message where $m^+ \geq \overline{\nu}_+$ and $m^- \geq 0$, follows a history at which the advisor stops as soon as $\nu_n^+ = m^+$, reports all positive outcomes and a subset of negative outcomes. Since it must be that the advisor prefers acceptance given $\nu_n^+ \geq \overline{\nu}_+$ by Claim 1b, reporting all positive outcomes is in line with $\underline{\sigma}_A^M$. Note that the advisor must deviate from $\underline{\sigma}_A^T$ to achieve $m^+ > \overline{\nu}_+$. The lowest number of deviations to give lead to $m^+ > \overline{\nu}_+$ arises if the advisor stops as soon as $\nu_n^+ = m^+$. Any deviation from $\underline{\sigma}_A^M$ to report some number of negative outcomes is equally likely. Hence, as $\varepsilon \to 0$, a message where $m^+ \geq \overline{\nu}_+$ and $m^- \geq 0$ arises if and only if at least m^+ and at most $N - m^-$ outcomes in N tests are positive. By the same reasoning, in the limit as $\varepsilon \to 0$, any message where $m^+ \geq 0$ and $m^- \geq \overline{\nu}_-$ is most likely to arise following a history in which the advisor stops testing as soon as $\nu_n^- = m^-$, reports all negative outcomes and some subset of positive outcomes. Hence, as $\varepsilon \to 0$, any message where $m^+ \geq 0$ and $m^- \geq \overline{\nu}_-$ arises if and only if at least m^- and at most $N - m^+$ outcomes in N tests are negative.

In the limit as $\varepsilon \to 0$, any message where $m^+ < \overline{\nu}_+$ and $m^- < \overline{\nu}_-$, is most likely to arise following a history in which the advisor stops testing as soon as $\nu_n^- = \overline{\nu}_-$, fails to report $\overline{\nu}_- - m^-$ negative outcomes and reports a subset of positive outcomes. To see why, suppose the advisor follows $\underline{\sigma}_A^T$. Then by period N, either $\nu_n^+ = \overline{\nu}_+$ or $\nu_n^- = \overline{\nu}_-$. If $\nu_n^+ = \overline{\nu}_+$, then to send any message where $m^+ < \overline{\nu}_+$ and $m^- < \overline{\nu}_-$ the advisor must omit $\overline{\nu}_+ - m^+$ positive outcomes, which occurs with a probability of $\varepsilon^{N\left(\overline{\nu}_+ - m^+\right)}$, and if $m^- > 0$ then he must report a subset of negative outcomes, which occurs with probability ε. If $\nu_n^- = \overline{\nu}_-$, then to send any message where $m^+ < \overline{\nu}_+$ and $m^- < \overline{\nu}_-$ the advisor must omit $\overline{\nu}_- - m^-$ negative outcomes, which occurs with a probability of $\varepsilon^{\left(\overline{\nu}_- - m^-\right)}$, and if $m^+ > 0$ then he must report a subset of positive outcomes, which occurs with probability ε. Since $N > \overline{\nu}_- - m^-$, it must be that $N\left(\overline{\nu}_+ - m^+\right) > \overline{\nu}_- - m^-$. Therefore, even if $m^- = 0$ and $0 < m^+ < \overline{\nu}_+$, it is more likely that $\nu_n^- = \overline{\nu}_-$ than $\nu_n^+ = \overline{\nu}_+$ since $\varepsilon^{(\overline{\nu}_- + 1)} > \varepsilon^{N\left(\overline{\nu}_+ - m^+\right)}$. In addition, a single deviation from $\underline{\sigma}_A^T$ occurs with probability ε^N, and therefore, is less likely than the event that the advisor followed $\underline{\sigma}_A^T$ and deviated from $\underline{\sigma}_A^M$. Hence, as $\varepsilon \to 0$ any message where $m^+ < \overline{\nu}_+$ and $m^- < \overline{\nu}_-$ arises if and only if at least $\overline{\nu}_-$ and at most $N - m^+$ outcomes in N tests are negative.

Proof Claim 2c: First, suppose that $max\left\{\widehat{\nu}_+, \overline{\nu}_+\right\} = \overline{\nu}_+$, which implies $min\left\{\overline{\nu}_-, N - \widehat{\nu}_+ + 1\right\} = \overline{\nu}_-$. The equilibrium is sequential by the same reasoning as in Claim 1d above.

Second, suppose that $max\left\{\widehat{\nu}_+, \overline{\nu}_+\right\} = \widehat{\nu}_+$, which implies $min\left\{\overline{\nu}_-, N - \widehat{\nu}_+ + 1\right\} = N - \widehat{\nu}_+ + 1$. Consider the following completely mixed strategy for the advisor with $\varepsilon \in (0, 1)$:

$$\underline{\sigma}_A^{T\varepsilon}\left(h_n\right) = \begin{cases} \varepsilon^N & \text{if } \nu_n^+ \geq \widehat{\nu}_+ \text{ or if } \nu_n^- \geq N - \widehat{\nu}_+ + 1, \\ 1 - \varepsilon^N & \text{otherwise,} \end{cases} \tag{106}$$

and $\underline{\sigma}_A^{M\varepsilon}$ is as follows: If $x_N \geq r + 1$ the advisor fails to report each positive outcome in $\underline{\sigma}_A^{T\varepsilon}$ independently with probability ε^N and reports any strictly positive number of negative outcomes with probability ε. If $x_N \leq r$ the advisor fails to report each negative outcome in $\underline{\sigma}_A^{T\varepsilon}$ independently with probability ε and reports any strictly positive number of positive outcomes with probability ε.

Clearly, $\left(\underline{\sigma}_A^{T\varepsilon}, \underline{\sigma}_A^{M\varepsilon}\right) \to \left(\underline{\sigma}_A^T, \underline{\sigma}_A^M\right)$ as $\varepsilon \to 0$. I show that the DM's beliefs also converge. In the limit as $\varepsilon \to 0$, any message where $m^+ \geq \widehat{\nu}_+$ and $m^- \geq 0$ arises following a history at which the advisor stopped as soon as $\nu_n^+ = m^+$, reported all positive outcomes and a subset of negative outcomes. Since it must be that the advisor prefers acceptance given $\nu_n^+ \geq \widehat{\nu}_+ \geq \overline{\nu}_+$ by Claim 1a, reporting all positive outcomes is in line with $\underline{\sigma}_A^M$. Note that the advisor must deviated from $\underline{\sigma}_A^T$ to achieve $m^+ > \widehat{\nu}_+$. The lowest number of deviations to lead to $m^+ \geq \widehat{\nu}_+$ arises if the advisor stopped as soon as $\nu_n^+ = m^+$. Any deviation from $\underline{\sigma}_A^M$ to report some number of negative outcomes is equally likely. Hence, as $\varepsilon \to 0$, any message where $m^+ \geq \widehat{\nu}_+$ and $m^- \geq 0$ arises if and only if at least m^+ and at most $N - m^-$ outcomes in N tests are positive. In addition, in the limit as $\varepsilon \to 0$, any message where $\widehat{\nu}_+ \geq m^+ \geq \overline{\nu}_+$ and $m^- \geq 0$ arises following a history at which the advisor stopped as soon as $\nu_n^- = N - \widehat{\nu}_+ + 1$, reported all positive outcomes and a subset of negative outcomes. No deviation from $\underline{\sigma}_A^T$ is necessary. Since it must be that the advisor prefers acceptance given $\nu_n^+ \geq \overline{\nu}_+$ by Claim 1a, reporting all positive outcomes is in line with $\underline{\sigma}_A^M$. Therefore, the only deviation is to report some number of negative outcomes if $m^- > 0$ and this kind of deviation is always necessary to generate a message with $m^+ > 0$ and $m^- > 0$. Hence, as $\varepsilon \to 0$, any message where $\widehat{\nu}_+ \geq m^+ \geq \overline{\nu}_+$ and $m^- \geq 0$ arises if and only if at least m^+ and at most $N - m^-$ outcomes in N tests are positive.

In the limit as $\varepsilon \to 0$, any message where $\overline{\nu}_- > m^- \geq N - \widehat{\nu}_+ + 1$ and $r + m^- + 1 \leq m^+ < \overline{\nu}_+$, follows a history in which the advisor stops testing as soon as $\nu_n^- = m^-$, reports all positive outcomes and some subset of negative outcomes. Note that the advisor must deviate from $\underline{\sigma}_A^T$ to achieve $m^- > N - \widehat{\nu}_+ + 1$. The lowest number of deviations to allow for $m^- > N - \widehat{\nu}_+ + 1$ arises if the advisor stopped as soon as $\nu_n^- = m^-$. Suppose the advisor preferred acceptance, i.e. $\nu_n^+ - \nu_n^- \geq r + 1$. Then the only necessary deviation from $\underline{\sigma}_A^M$ is that a subset of negative outcomes is reported. If the advisor had preferred rejection then he would need to deviate from $\underline{\sigma}_A^M$ by failing to report some of the positive outcomes as well as reporting some subset of negative outcomes. Hence, as $\varepsilon \to 0$, this message arises if and only if at least m^- and at most $N - m^+$ outcomes in N tests are positive. In addition, in the limit as $\varepsilon \to 0$, any message where $m^- \geq \overline{\nu}_-$ (which implies $m^+ < \overline{\nu}_+$) or where $\overline{\nu}_- > m^- \geq N - \widehat{\nu}_+ + 1$ and $m^+ < min\left\{\overline{\nu_n^+}, r_A + m^- + 1\right\}$, follows a history in which the advisor stops testing as soon as $\nu_n^- = m^-$, reports all negative outcomes and some subset of positive outcomes. Again, the lowest number of deviations to allow for $m^- > N - \widehat{\nu}_+ + 1$ arises if the advisor stopped as soon as $\nu_n^- = m^-$. If $m^- \geq \overline{\nu}_-$ or if $\overline{\nu}_- > m^- \geq N - \widehat{\nu}_+ + 1$ and $m^+ - m^- \leq r$, the only necessary deviation from $\underline{\sigma}_A^M$ is that a subset of positive outcomes is reported and this kind of deviation is always necessary to generate a message with $m^+ > 0$ and $m^- > 0$. Hence, as $\varepsilon \to 0$ this message arises if and only if at least m^- and at most $N - m^+$ outcomes in N tests are negative.

Any message where $m^+ < \overline{\nu}_+$ and $N - \widehat{\nu}_+ + 1 > m^-$, in the limit as $\varepsilon \to 0$, follows a history in which the advisor stops testing as soon as $\nu_n^- = N - \widehat{\nu}_+ + 1$, fails to report $N - \widehat{\nu}_+ + 1 - m^-$

negative outcomes and reports some subset of positive outcomes. Suppose the advisor follows $\underline{\sigma}_A^T$. Then by period N either $\nu_n^- = N - \widehat{\nu}_+ + 1$ or $\nu_n^+ = \widehat{\nu}_+$. If $\nu_n^+ = \widehat{\nu}_+$ then the advisor prefers acceptance and failed to report $\widehat{\nu}_+ - m^+$ positive outcomes, which happens with probability $\varepsilon^{N\left(\widehat{\nu}_+ - m^+\right)}$. If $\nu_n^- = N - \widehat{\nu}_+ + 1$ then if he preferred acceptance he failed to report $\nu_n^+ - m^+$ positive outcomes where $\nu_n^+ < \widehat{\nu}_+$, which happens with probability $\varepsilon^{N\left(\nu_n^+ - m^+\right)}$, or if he preferred rejection and failed to report $N - \widehat{\nu}_+ + 1 - m^-$ negative outcomes, which happens with probability $\varepsilon^{\left(N - \widehat{\nu}_+ + 1 - m^-\right)}$. Since $N - \widehat{\nu}_+ + 1 - m^- > N\left(\nu_n^+ - m^+\right) > N\left(\widehat{\nu}_+ - m^+\right)$, it is most likely that $\nu_n^- = N - \widehat{\nu}_+ + 1$ and the advisor fails to report $N - \widehat{\nu}_+ + 1 - m^-$ negative outcomes. In addition, a single deviation from $\underline{\sigma}_A^T$ occurs with probability ε^N, and therefore, is less likely than the event that the advisor followed $\underline{\sigma}_A^T$ and deviated from $\underline{\sigma}_A^M$. Hence, as $\varepsilon \to 0$ this message arises if and only if at least $N - \widehat{\nu}_+ + 1$ and at most $N - m^+$ outcomes in N tests are negative.

B.4 Proposition 7 [Simultaneous Testing]

Consider the following modification to games Γ and Γ^o. The advisor chooses the number of tests ex ante, i.e. his testing strategy is $\sigma_A^T : H_0 \to \{0, ..., N\}$ in Γ and $\sigma_A^o : H_0 \to \{0, ..., N\}$ in Γ^o. The advisor chooses the number of tests at h_0 to maximize his expected payoff given his beliefs μ_A and the DM's strategy. In Γ^o, $\frac{\mu(h_n)}{1-\mu(h_n)} = l\left(x_n\right)$ and, in Γ, $\frac{\mu_A(h_n)}{1-\mu_A(h_n)} = l\left(x_n\right)$ for $n \in \{0, N\}$ where x_n is the number of excess positive outcomes in h_n. Denote the likelihood ratio at the end of period n by l_n.

Part 1. For the skepticism effect to exist, it must be that the DM prefers acceptance when all N tests are positive, i.e. $\lambda_{DM} \leq l\left(N\right)$, otherwise, $\sigma_{DM}^o = reject$ for any h_N and $\sigma_{DM} = reject$ for any m. Suppose testing is observable, and r_A and r_{DM} are defined by

$$r_{DM} \equiv sup\left\{j | j \in \{0, ..., N-1\}, l\left(j\right) < \lambda_{DM}\right\}, \quad (107)$$

$$r_A \equiv sup\left\{j | j \in \mathbb{Z}, l\left(j\right) < \lambda_A\right\}, \quad (108)$$

and $r_A + 1 \leq r_{DM}$. I show that if r_{DM} is even (odd) the advisor chooses the largest odd (even) number of tests n that satisfies $n \leq N$. Hence, given any even (odd) r_{DM}, if N is odd (even) the advisor chooses $n = N$ and, therefore, the DM can never be strictly better off under hidden testing.

First, I show that if r_{DM} is even (odd) then the advisor never chooses an even (odd) number of tests. Suppose the advisor has already committed to run n tests, where $n \leq N - 1$. An additional test only affects the advisor's expected payoff if it affects the DM's choice. There are two situations in which the additional test affects the DM's choice. The first situation is that after n tests the DM rejects, but if the additional test is positive, she accepts. This situation arises if and only if $l_n = l(r_{DM})$. The second situation is that after n tests the DM accepts, but if the additional test is negative, she rejects. This situation arises if and only if $l_n = l(r_{DM}+1)$. Since the advisor prefers accept if at $l_n = l(r_{DM})$, he is strictly better off with an additional test in the first situation, but strictly worse off in the second situation. If r_{DM} is even (odd), then the first situation can only arise if n is even (odd) and the second only if n is odd (even). Hence, there is only an upside to an additional test if n is even (odd) and only a downside if n is odd (even). Therefore, the advisor strictly prefers to run an additional test after an even (odd) number of tests.

Consider r_{DM} is even (odd) and n is odd (even). Next, I show that if the advisor benefits from running an additional two tests at some n then he benefits from running the largest odd (even) number of tests such that $n \leq N$. There are two situations in which the additional two tests could affect the DM's choice. The first situations is that after n tests the DM rejects, but if both additional tests are positive, she accepts. This situation arises if and only if $l_n = l(r_{DM}-1)$, i.e. at a total of $\nu_n^+ = \frac{n+(r_{DM}-1)}{2}$ positive outcomes. The second situation is that after n tests the DM accepts, but if both additional tests are negative, she rejects. This situation arises if and only if $l_n = l(r_{DM}+1)$, i.e. at a total of $\nu_n^+ = \frac{n+r_{DM}+1}{2}$ positive outcomes. The advisor is strictly better off by conducting these test if and only if

$$\begin{aligned}\lambda_A Pr(false)\Bigg[& Pr\left(\nu_n^+ = \frac{n+r_{DM}+1}{2}|false\right)p^2 - \\ & Pr\left(\nu_n^+ = \frac{n+(r_{DM}-1)}{2}|false\right)(1-p)^2\Bigg] \\ +Pr(true)\Bigg[& Pr\left(\nu_n^+ = \frac{n+(r_{DM}-1)}{2}|true\right)p^2 \\ & -Pr\left(\nu_n^+ = \frac{n+r_{DM}+1}{2}|true\right)(1-p)^2\Bigg] > 0 \end{aligned} \tag{109}$$

where

$$Pr\left(\nu_n^+ = j|true\right) = \binom{n}{j}(1-p)^{n-j}p^j, \tag{110}$$

$$Pr\left(\nu_n^+ = j|false\right) = \binom{n}{j}(1-p)^j p^{n-j}. \tag{111}$$

Substituting into (109):

$$\frac{n!\lambda_A (1-q)(1-p)^{\frac{n+r_{DM}+1}{2}} p^{\frac{n-(r_{DM}-1)}{2}} 2}{\left(\frac{n+r_{DM}+1}{2}-1\right)!\left(\frac{n-r_{DM}-1}{2}\right)!}\left[\frac{p}{(n+r_{DM}+1)}-\frac{(1-p)}{(n-r_{DM}+1)}\right]$$
$$+\frac{n!qp^{\frac{n+r_{DM}+1}{2}}(1-p)^{\frac{n-r_{DM}+1}{2}} 2}{\left(\frac{n+r_{DM}+1}{2}-1\right)!\left(\frac{n-r_{DM}+1}{2}\right)!}\left[\frac{p}{n-r_{DM}+1}-\frac{1-p}{n+r_{DM}+1}\right]>0 \Leftrightarrow$$
$$\lambda_A (1-q)(1-p)^{r_{DM}}\left[(n-r_{DM}+1)p-(n+r_{DM}+1)(1-p)\right]$$
$$+qp^{r_{DM}}\left[(n+r_{DM}+1)p-(n-r_{DM}+1)(1-p)\right]>0. \qquad (112)$$

The LHS increases with n since $p>1-p$. Hence, if (112) holds at some n then it must hold at any larger n.

What is left to show is that the advisor strictly prefers to choose an odd (even) number of tests large enough that it is possible that the outcomes lead the DM to accept. The advisor is indifferent between running $r_{DM}-1$ or fewer tests, because even if all $r_{DM}-1$ tests were positive, the DM would still reject. However, he must strictly benefit from running $r_{DM}+1$ tests, because if the DM accepts than the advisor also prefers to accept. By the argument above, if the advisor benefits from an additional two tests at $n=r_{DM}-1$ then he must benefit from running two additional tests at any larger n. Hence, the advisor chooses the largest odd (even) number of tests feasible.

Part 2. Suppose $l(0)<\lambda_{DM}\leq l(1)$ and $l(N-1)<\lambda_A\leq l(N)$. I show that, under observable testing, the advisor does not to conduct any tests. First, I show that if r_{DM} is even (odd), the advisor never chooses an odd (even) number of tests. There are the two situations in which the additional test affects the DM's choice, as described in Part 1. The advisor is strictly better off with an additional test if $l_n=l(1)$, but strictly worse off if $l_n=l(0)$. Given r_{DM} even (odd), $l_n=l(1)$ can only arise if n is odd (even) and $l_n=l(0)$ only if n is even (odd). Hence, there is only an upside to an additional test if n is odd (even) and only a downside if n is even (odd). Therefore, the advisor strictly prefers to run an additional test after an odd number of tests. Second, I show that if the advisor benefits from running an additional two tests at some n then he benefits from running the largest even number of tests such that $n\leq N$. Consider only even n. There are two situations in which the additional two tests could affect the DM's choice, as described in Part 1. The first situation arises if and only if $l_n=l(0)$. The second situation arises if and only if $l_n=l(2)$.

The advisor is strictly better off by conducting two additional tests if and only if

$$\begin{aligned}
&\lambda_A Pr\left(false\right)\left[Pr\left(\nu_n^+ = \frac{n+2}{2}|false\right)p^2 - Pr\left(\nu_n^+ = \frac{n}{2}|false\right)\left(1-p\right)^2\right] \\
&\quad +Pr\left(true\right)\left[Pr\left(\nu_n^+ = \frac{n}{2}|true\right)p^2 - Pr\left(\frac{n+2}{2}|true\right)\left(1-p\right)^2\right] > 0 \Leftrightarrow \\
&\quad \frac{n!\lambda_A\left(1-q\right)\left(1-p\right)^{\frac{n+2}{2}}p^{\frac{n}{2}}2}{\left(\frac{n+2}{2}-1\right)!\left(\frac{n-2}{2}\right)!}\left[\frac{p}{\left(n+2\right)} - \frac{1-p}{n}\right] \\
&\quad + \frac{n!qp^{\frac{n+2}{2}}\left(1-p\right)^{\frac{n}{2}}2}{\left(\frac{n+2}{2}-1\right)!\left(\frac{n}{2}\right)!}\left[\frac{p}{n} - \frac{1-p}{n+2}\right] > 0 \Leftrightarrow \\
&\quad \lambda_A\left(1-q\right)\left(1-p\right)\left[np - \left(n+2\right)\left(1-p\right)\right] \\
&\quad +qp\left[\left(n+2\right)p - \left(n\right)\left(1-p\right)\right] > 0. \qquad (113)
\end{aligned}$$

The LHS increases with n since $p > 1-p$. Hence, if (113) holds at some n then it must hold at any larger n. Therefore, if the advisor runs any tests at all then he runs all N tests. Lastly, the advisor prefers no test to N tests if and only if

$$\begin{aligned}
&Pr\left(true\right) < Pr\left(false\right)\lambda_A Pr\left(\nu_n^+ > \frac{N}{2}|false\right) + Pr\left(true\right)Pr\left(\nu_n^+ \leq \frac{N}{2}|true\right) \Leftrightarrow \\
&\lambda_A > \frac{qPr\left(\nu_n^+ > \frac{N}{2}|true\right)}{\left(1-q\right)Pr\left(\nu_n^+ > \frac{N}{2}|false\right)}. \qquad (114)
\end{aligned}$$

This must hold since for $N > 2$,

$$\frac{qPr\left(\nu_n^+ > \frac{N}{2}|true\right)}{\left(1-q\right)Pr\left(\nu_n^+ > \frac{N}{2}|false\right)} < l\left(N-1\right) \equiv \frac{qp^{N-1}}{\left(1-q\right)\left(1-p\right)^{N-1}} < \lambda_A. \qquad (115)$$

Finally, suppose testing is hidden and the advisor runs all N tests. By the same argument as in Lemma 7, there is an advisor-preferred equilibrium in which the advisor discloses all positive outcomes if and only if $l_N = l\left(N\right)$ since $l\left(N-1\right) < \lambda_A \leq l\left(N\right)$, and otherwise, discloses only negative outcomes. The DM acts in line with the advisor's interest for any on-path message, by the same reasoning as in Lemma 7. Therefore, the advisor indeed runs all N tests. Hence, the DM must be strictly better off under hidden testing.

B.5 Proposition 8 [Infinite Horizon]

Consider the following modification to games Γ and Γ^o. There are infinitely many discrete periods, $n = 0, 1, ...$ The advisor incurs a cost $c > 0$ for each test. In Γ^o, in each period n, the advisor chooses to test or not, but once he has stopped testing he cannot test again. When the advisor has stopped, the DM chooses an action, i.e. $\sigma^o_{DM} : H_n \longrightarrow \{accept, reject\}$. In Γ, in each period n, the advisor privately chooses to test or not and when he stops he sends a message $m \in \mathcal{M}$, where $\mathcal{M}_n$ as in (4) and $\mathcal{M} = \cup_{n=0}^{\infty}\mathcal{M}_n$. Given the unordered history of outcomes $\widetilde{h}_n$ at the end of period n, the set of feasible messages is $M\left(\widetilde{h}_n\right) = \mathcal{P}\left(\widetilde{h}_n\right)$. Then the DM chooses $a \in \{accept, reject\}$. The DM does not know in which period the message was sent. A reporting strategy for the advisor is $\sigma^M_A : H_n \longrightarrow \mathcal{M}_n$ for $n = 0, 1, ...$ A strategy for the DM is $\sigma_{DM} : \mathcal{M} \longrightarrow \{accept, reject\}$. In each scenario, the solution concept is a PBE.

Insurance Effect: Suppose $\lambda_A > \lambda_{DM}$. Under hidden testing, once the advisor has stopped testing at some h_n, the DM acts in his interest given h_n, by the same reasoning as in the proof of Lemma 7. Therefore, the advisor's optimal testing strategy is the one chosen if the advisor was in charge of taking the final decision. Hence, the advisor's optimal testing strategy is time-independent. In order for the DM to be better off under hidden than observable testing, it is necessary that with some probability the advisor finds x excess positive outcomes such that he prefers acceptance, i.e. $l(x) \geq \lambda_A$. This is because then the DM prefers acceptance, since $l(x) \geq \lambda_A > \lambda_{DM}$, and acceptance is chosen in equilibrium. To be able to find such outcomes, the advisor needs to test at any history where the number x of excess positive outcomes satisfies $l(0) < l(x) \leq \lambda_A$, i.e. for any x satisfying $l(0) < l(x) \leq \lambda_A$ the advisor's continuation value of testing must exceed his continuation value of stopping.

Under observable testing, the DM's optimal strategy is to reject if and only if $l(x_n) < \lambda_{DM}$, where x_n denotes the number of excess positive outcomes when the advisor stops testing. Therefore, the advisor's optimal testing strategy is again time-independent. His value of stopping at h_n is the same as under hidden testing if $l(x_n) < \lambda_{DM}$ or $l(x_n) \geq \lambda_A$, because then the DM and the advisor agree on the optimal final decision. However, it is lower if $\lambda_{DM} \leq l(x_n) < \lambda_A$. Therefore, if the advisor does not test at any history h_n such that $l(0) < \lambda_{DM} < l(x_n) \leq \lambda_A$ when testing is observable, then he also does not test at any such h_n when testing is hidden. Hence, the insurance effect cannot exist.

Skepticism Effect: Let $\widetilde{h}_n = (\nu^+_n, \nu^-_n)$ denote the history at the end of period n with ν^+_n positive and ν^-_n negative outcomes. Define two thresholds of c:

$$\overline{c} = \min\left\{\frac{q(1-p)p}{q(1-p)+(1-q)p}, \frac{q(3p^2-3p^3+p^4)}{2-p^3-q+3pq-3p^2q+2p^3q}\right\}, \tag{116}$$

$$\underline{c} = \max\left\{\frac{q(1-p)}{2(q(1-p)+(1-q)p)}, \frac{q(1-p)^2}{q(1-p)^2+(1-q)p^2}\right\}. \tag{117}$$

The skepticism effect exists if c satisfies $\overline{c} > c > \underline{c}$, $\lambda_A = 0$ and

$$\frac{q\left(1-p^2\right)}{\left(1-q\right)\left(1-\left(1-p\right)^2\right)} < \lambda_{DM} < \frac{qp\left(p+\left(1-p\right)^2+p\left(1-p\right)^2\right)}{\left(1-q\right)\left(1-p\right)\left(1-p+p^2+p^2\left(1-p\right)\right)}. \tag{118}$$

I show that for any q, there exists a $p \in \left(\frac{1}{2},1\right)$ such that $\overline{c} > \underline{c}$. At $p=1$, $\overline{c} = \underline{c} = 0$. As $p \to 1$, $\underline{c} = \frac{q(1-p)p^2}{2q(1-p)+2(1-q)p}$ and $\overline{c} = \frac{q(1-p)p}{q(1-p)+(1-q)p}$. As $p \to 1$, $\frac{\partial \overline{c}}{\partial p} < 0$ and $\frac{\partial \underline{c}}{\partial p} < 0$ but $-\frac{\partial \overline{c}}{\partial p} > -\frac{\partial \underline{c}}{\partial p}$. Hence, $\overline{c} > \underline{c}$ for p sufficiently large.

Under observable testing, $\underline{\sigma}_{DM}\left(h_n\right) = accept$ if and only if $l_n \geq l\left(1\right)$, since

$$l\left(0\right) < \lambda_{DM} < \frac{qp\left(p+\left(1-p\right)^2+p\left(1-p\right)^2\right)}{\left(1-q\right)\left(1-p\right)\left(1-p+p^2+p^2\left(1-p\right)\right)} < l\left(1\right). \tag{119}$$

The advisor's optimal strategy is time-independent. $\underline{\sigma}_A\left(h_n\right) = 0$ if $l_n \geq l\left(1\right)$, since he prefers acceptance independent of the state and the DM accepts. In addition, $\underline{\sigma}_A\left(h_n\right) = 0$ if $l_n \leq l\left(-1\right)$ since the expected cost of testing outweighs the expected benefit. In particular, he would stop at $l_n = l\left(-1\right)$ even if the next two outcomes were known to be positive and, hence, would lead the DM to accept since $c > \underline{c}$ and

$$-2c < -Pr\left(true|\widetilde{h}_n = \left(0,1\right)\right) = -\frac{q\left(1-p\right)}{q\left(1-p\right)+\left(1-q\right)p}. \tag{120}$$

Given that $\underline{\sigma}_A\left(h_n\right) = 0$ if either $l_n \geq l\left(1\right)$ or $l_n \leq l\left(-1\right)$, $\underline{\sigma}_A\left(h_n\right) = 1$ if $l_n = l\left(0\right)$ since $c < \overline{c}$ and $-q\left(1-p\right) - c > -q \Leftrightarrow qp > c$.

Next, suppose testing is hidden. I show that it cannot be part of an equilibrium that $\underline{\sigma}_{DM}\left(m\right) = accept$ for $m = \left(m^+, 0\right)$ where $m^+ \geq 1$. To see why, suppose this was the case. Then for any $\widetilde{h}_n$ where $\nu_n^+ \geq 1$, the advisor reports only positives, i.e. $\underline{\sigma}_A^M\left(\widetilde{h}_N\right) = \left(\nu_N^+, 0\right)$. Hence, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 0$ if $\nu_n^+ \geq 1$. In addition, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 0$ if $\widetilde{h}_n = \left(0,2\right)$, because even if the next test were known to be positive, the advisor is not willing to pay since $c > \underline{c}$ and

$$-c < -Pr\left(true|\widetilde{h}_n = \left(0,2\right)\right) = -\frac{q\left(1-p\right)^2}{q\left(1-p\right)^2+\left(1-q\right)p^2}. \tag{121}$$

But the advisor optimally continues testing at $\widetilde{h}_n = \left(0,1\right)$, since $c < \overline{c}$ and

$$-Pr\left(true|\widetilde{h}_n = \left(0,1\right)\right)\left(1-p\right) - c > -Pr\left(true|\widetilde{h}_n = \left(0,1\right)\right) \Leftrightarrow$$
$$\frac{q\left(1-p\right)p}{q\left(1-p\right)+\left(1-q\right)p} > c. \tag{122}$$

Given (122), the advisor also tests at the prior since his belief that the state is true is then even higher. The advisor's strategy means that the DM receives $m = (m^+, 0)$ where $m^+ \geq 1$ if either $\widetilde{h}_1 = (1,0)$ or $\widetilde{h}_2 = (1,1)$. In equilibrium, the DM must reject such a report since $\frac{q\left(1-p^2\right)}{(1-q)\left(1-(1-p)^2\right)} < \lambda_{DM}$.

Next, I show that there exists an equilibrium in which $\underline{\sigma}_{DM}(m) = accept$ if and only if $m = (m^+, 0)$ where $m^+ \geq 2$. Suppose the DM followed this strategy. Then for any $\widetilde{h}_n$ where $\nu_n^+ \geq 2$, the advisor reports only positives, i.e. $\underline{\sigma}_A^M\left(\widetilde{h}_n\right) = (\nu_n^+, 0)$. Hence, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 0$ if $\widetilde{h}_n$ where $\nu_n^+ \geq 2$. In addition, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 0$ if $\widetilde{h}_n = (0,1)$, because even if the next two tests were known to be positive, the advisor is not willing to test since $c > \underline{c}$ and (120). In addition, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 0$ if $\widetilde{h}_n = (1,3)$, because even if the next test were known to be positive, he is not willing to test since $c > \underline{c}$ and $Pr\left(true|\widetilde{h}_n = (0,2)\right) = Pr\left(true|\widetilde{h}_n = (1,3)\right)$ and (121). However, $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 1$ if $\widetilde{h}_n = (1,2)$ since $Pr\left(true|\widetilde{h}_n = (1,2)\right) = Pr\left(true|\widetilde{h}_n = (0,1)\right)$ and (122). Suppose $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 1$ if $\widetilde{h}_n = (1,1)$ or if $\widetilde{h}_n = (1,0)$. Then, $\underline{\sigma}_A^T(h_n) = 1$ if $\widetilde{h}_n = (0,0)$ since $\overline{c} > c$ and

$$\begin{aligned} -q\left(1-p+p(1-p)^3\right) - \left(q(1-p) + (1-q)p\right)c \\ -\left(qp^2 + (1-q)(1-p)^2\right)2c - \left(q(1-p)p^2 + (1-q)(1-p)^2 p\right)3c \\ -\left(q(1-p)^2 p + (1-q)(1-p)p^2\right)4c > -q \\ \Leftrightarrow \\ \frac{q\left(3p^2 - 3p^3 + p^4\right)}{2 - p^3 - q + 3pq - 3p^2 q + 2p^3 q} > c \end{aligned} \tag{123}$$

Given (123), it must also be that $\underline{\sigma}_A^T\left(\widetilde{h}_n\right) = 1$ if $\widetilde{h}_n = (1,1)$ or if $\widetilde{h}_n = (1,0)$ since he only needs one additional positive outcome for acceptance and his posterior belief of the state being true is at least as high as when $\widetilde{h}_n = (0,0)$. This implies that $m = (m^+, 0)$ where $m^+ \geq 2$ is sent if and only if $h_2 = (+,+)$ or $h_3 = (+,-,+)$ or $h_4 = (+,-,-,+)$. In equilibrium, the DM accepts conditional on $m = (2,0)$ since

$$\lambda_{DM} \leq \frac{\underline{\mu}_{DM}(m = (2,0))}{1 - \underline{\mu}_{DM}(m = (2,0))} = \frac{qp\left(p + (1-p)^2 + p(1-p)^2\right)}{(1-q)(1-p)\left(1-p+p^2+p^2(1-p)\right)}. \tag{124}$$

Recall that the advisor stops testing in period 4 or earlier on the equilibrium path. Denote the set of lists of Nature's first four draws of outcomes by

$$\Phi_4 \equiv \left\{(s_1, ..., s_4) \,|\, s_i \in \{-,+\}, i \in \{1, ..., 4\}\right\}. \tag{125}$$

Denote the subset of Φ_4 for which acceptance is chosen under hidden and observable testing by Φ^o and Φ^h respectively, where

$$\Phi^o = \{(s_1, ..., s_4)\,|s_1 = (+)\}, \tag{126}$$

$$\Phi^h = \{(s_1, ..., s_4)\,|s_1 = (+), s_2 = (+) \vee s_3 = (+) \vee s_4 = (+)\}. \tag{127}$$

Therefore, the only list ϕ which satisfies $\phi \in \Phi^o$ and $\phi \notin \Phi^h$ is $\phi = (+, -, -, -)$ and, given this list, the DM prefers rejection since $l(-2) < \lambda_{DM}$. Hence, the DM is better off under hidden testing.